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DESIGN GUIDE

UNITED STATES ARMY

SHOCK AND VIBRATION
TECHNICAL DESIGN GUIDE

BOOK 1 OF 2

- VOLUME I – METHODOLOGY AND DESIGN PHILOSOPHY
- VOLUME II – ANALYTICAL PROCEDURES

SHOCK AND VIBRATION
TECHNICAL DESIGN GUIDE

U.S. Army Electronics Command
Mechanical Engineering Branch
Fort Monmouth, New Jersey

Environmental Engineering Department
Technical Services Group Office
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Hughes Aircraft Company
Fullerton, California

Submitted by:

[signature]
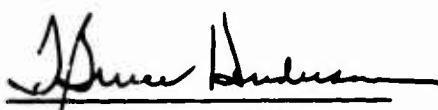
T. Bruce Henderson
Program Manager

Signal Corps
Technical Requirements
SCL-7851A

Contract No. DAAB07-67-C-0111
Hughes Document No. FR68-10-671

FOREWORD

This Design Guide resulted from a study program conducted by the Environmental Engineering Department of Hughes-Fullerton, Hughes Aircraft Company, Fullerton, California, from October 1966 through July 1968.

This technical effort was performed for the Mechanical Engineering Branch, United States Army Electronics Command, Fort Monmouth, New Jersey, under contract number DAAB07-67-C-0111. The USAECOM Project Engineer was Mr. J.J. Oliveri, AMSEL/RD-GDM, Fort Monmouth, New Jersey.

This Design Guide is one in a series of guidebooks directed toward the problems of optimum packaging of Army electronic equipment and other sensitive equipment elements constrained to survive the Army field environments. The first in this series was a guidebook entitled "Optimum Mechanical Packaging of Electronic Equipment," developed at Hughes under contract to USAECOM. The "Shock and Vibration Technical Design Guide" uses the basic equipment categorizations developed during the previous study, and is directed toward the specific problem of dynamic structural integrity in Army Equipment packages.

The Program Manager at Hughes for this development effort was T.B. Henderson of the Environmental Engineering Department. Other people who made significant technical contributions to this Design Guide were Dr. Kenneth Foster, Consulting Dynamics Engineer; Professor R.E. Little of the University of Michigan; and R.H. Chrystie, M.A. Merrigan, G.M. Pomonik, and G. vonKampen of Hughes. Technical direction during the program effort is acknowledged from A.H. Jones and R.E. Freeman of Hughes and E. Laboissonniere, S. Adler, and P. Devreotes of USAECOM.
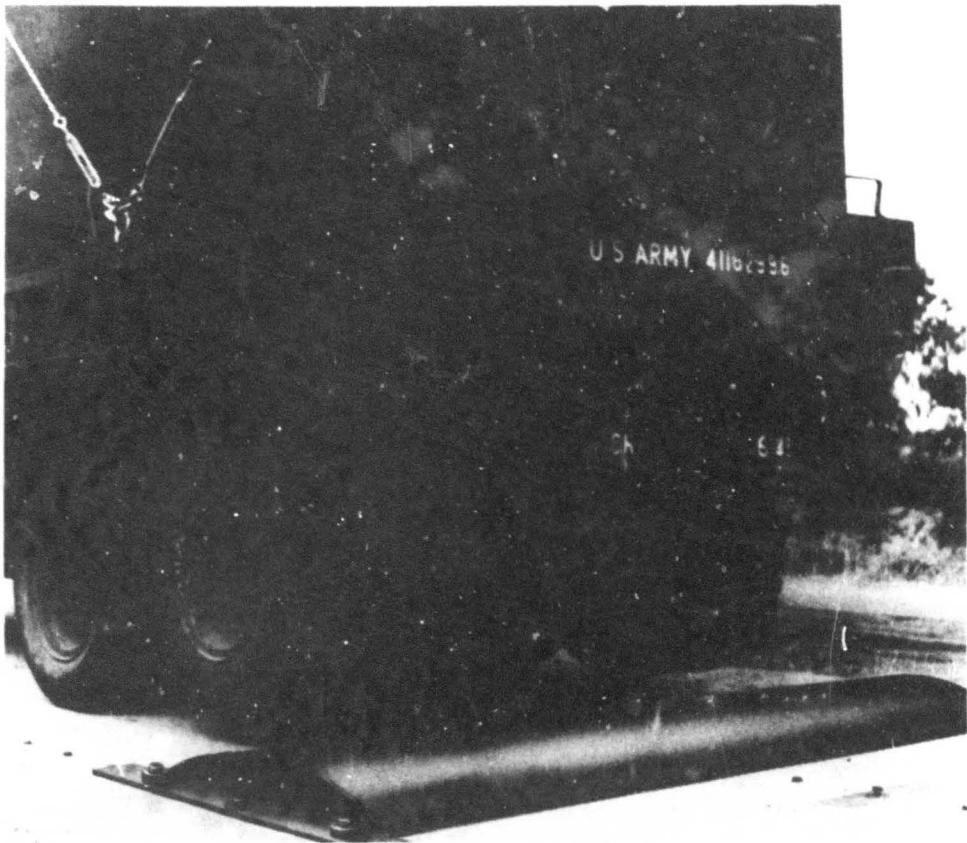
DYNAMIC STRUCTURAL INTEGRITY: Shock and vibration experiences provide the most rigorous test of structural reliability in Army equipment packages. These critical structural requirements are typified by the Road Mobility Tests over the Munson Course.

PROGRAM SUMMARY

This Design Guide has been developed in response to the technical requirements described in the Statement of Work, SCL-7851A, governed by Contract No. DAAB07-67-C-011?., issued by the United States Army Electronics Command, Fort Monmouth, New Jersey.

The objective of this Design Guide is to provide a useful summary of the language and technical disciplines peculiar to the Army shock and vibration requirements and their design impact on the equipment packages constrained to operate and survive this environment. This Design Guide is intended to provide the Mechanical Project Engineer with a reference on the technical aspects of the shock and vibration criteria. It is assumed that the reader has attained a reasonable level of knowledge in structural mechanics, yet is not an expert in the field of dynamics. The application of the theory and approaches outlined in the Design Guide will thus enable the Project Engineer to evaluate basic dynamic design situations that arise in the development of his equipment system.

Although the Design Guide is written with the Mechanical Project Engineer in mind, it has the secondary objective of acquainting other technical specialists with Army shock and vibration requirements and their relation to structural reliability in Army equipment systems. Accordingly, it is hoped that mechanical design people will find general utility in the Guidebook.

BOOK ABSTRACT

Volume I, Methodology and Design Philosophy, outlines the orderly approach
that the Mechanical Engineer will logically follow in accomplishing the structural
reliability of an Army equipment package. This volume also defines the dynamic
terminology needed to address the design problem, provides a background for
considerations and tradeoffs of importance to the Mechanical Engineer, and out-
lines a technique for effective use of the presented material.

Volume II, Analytical Procedures, presents a straightforward quantitative approach
to the evaluation of dynamic strength in an equipment package. The procedures are
based upon analytical definition of input excitations to equipment structures and
the development of transfer functions relating the response of the equipment
element to the excitation. The resulting static equivalent response may then be
compared with the allowable fragility level of the element under consideration, and
a margin of safety calculated.

Volume III, Related Technologies, is a compendium of the mechanical disciplines
needed by the Equipment Packaging Engineer in solving structural problems assoc-
iated with dynamic environments. This section details design aids and procedures
useful to the Structural Engineer. The information presented includes disserta-
tions on analytical tools and procedures, validation procedures, and hard-
ware oriented disciplines.

VOLUME I
METHODOLOGY AND DESIGN PHILOSOPHY

VOLUME I
METHODOLOGY AND DESIGN PHILOSOPHY

ABSTRACT:

The intent of this Volume is to provide an overview to the Mechanical
Engineer on the approach he will use in designing dynamic structural
integrity into the equipment package under consideration. The elements
of this general discussion include:

- An introduction to the subject of structural dynamics, the
  characterstics of a shock or vibration excited system, and a
  summary of the design choices available to the designer.

- A discussion of the Army transport methods employed in the field,
  how they relate to the Army equipment class categories, how the
  equipment class is used to define Quality Assurance tests, and
  ultimately the input loading criteria for the equipment system.

- A design approach to structural reliability which is supported by
  the Design Guide, including some ground rules for tailoring
  natural frequency and damping to optimize the inherent dynamic
  resistance of the equipment package.

- A roadmap through the Design Guide, and a qualitative overview of
  the design relevant information contained in Volume III and the
  application and limitations of the analytical procedures outlined
  in Volume II.

A

Volume I - Methodology and Design Philosophy

ERRATA SHEET

Page	Paragraph	Line	Correction
1-0	2	1	often
1-4	5	10	analyzing
1-6	3	2	approximation
1-7	Caption		...for representation of a complex system.
2-0	4	5	The
2-0	6	4	Class VI
2-3	5	1	...installed in or ...
2-6	2(chart)	2	Tracked Vehicles (T116)
3-0	2	7	discover
3-0	6	3-4	Dynamic Simulation
3-5	2 (#8)	2	rugged
3-5	7 (#3)	1	criterion
3-10	3	8	...class IV and V ...
4-0	2	9	analytical
4-0	2	16	analytical
4-4	Thesis	1	...which defines a ...
4-10	1	1	outlined
4-10	5	1	...factor is borderline,
4-11	R.H. Block	3	...Hard-Mount...
4-12	5	9	analytical
4-14	2	5	analytical
4-15	3	1	...are discussed,
4-15	Caption	1	...is presented...
5-8	Top caption	3	to operate while...
5-15	Caption	2	...and take full ...

VOLUME I
METHODOLOGY AND DESIGN PHILOSOPHY

CONTENTS

CONTENTS (continued)

VOLUME I

METHODOLOGY AND DESIGN PHILOSOPHY

SECTION 1 - INTRODUCTION

- Dynamic Structural Integrity of Electronic Equipment Packages

- The Characteristics of a Vibrating System

- Shock Effects in Equipment Structure

- The Single Degree-of-Freedom Concept

- Control of the Dynamic Environment

DYNAMIC STRUCTURAL INTEGRITY OF ELECTRONIC EQUIPMENT PACKAGES

**A balance between environmental stress and structural adequacy is the best approach
to dynamic integrity.**

The structural necessities imposed upon Army electronic equipment elements
by shock and vibration requirements are the most severe and demanding of
all the mechanical disciplines needed to accomplish the packaging task.
There is probably more material available in the technical literature on
dynamics and less basic understanding of the phenomena on the part of the
Mechanical Packaging Engineer than all the rest of his requirements. This
occurs in part as a result of the complexity of the dynamic sciences, and
part from a communications gap between the people doing the applied
research, and the people performing the package design function. This
design guide will attempt to bridge some of this knowledge gap by out-
lining a design methodology, an analysis procedure, and a compendium of
related mechanical disciplines.

In the past, a designer could often afford the luxury of ultra-conservative
loading criteria, since most of the mechanical systems were ground mob-
ilized, and hence had an abundance of power available for the task.
Modern transport modes by aircraft and helicopter have placed increased
emphasis on weight and bulk of Army equipment packages. The sledge-hammer
approach to support structure is no longer an acceptable alternative; all
this coupled with higher reliability constraints, two trends which are
divergent.

Balanced design is the proper correlation between strength and stress.
As the strength side of the equation is reduced with minimal safety mar-
gin (a necessary consequence of reduced bulk), closer definition of the
structural integrity of the package is then required to proportionately
reduce the pad on input loads. This is another way of saying that more
rigorous analytical techniques are indicated to exercise the structure at
higher stress levels, while maintaining a high level of reliability.

Dynamic structural integrity, as the term is used in this design guide,
is the inherent ability of the equipment support structure to sustain the
stress induced by shock and vibration loadings. The approach to the
problem of insuring integrity entails an understanding of the severity of
the input excitations, the mechanisms of energy transfer through the
structure, the material characteristics and fabrication processes which
make up the structural package, and a variety of other mechanical disci-
plines which may enhance the capability of the structure.

The analytical procedures outlined in Volume II are methods for numerically
evaluating structural integrity. The approach is based upon the premise
that static equivalent acceleration may be calculated from input loads
and dynamic transfer characteristics. This effective load may then be
compared with element strength (fragility) in the frequency domain, and
a factor of safety calculated. Thus, the value decision made as a result
of this analysis is based upon numbers, from which a numerical confidence
may be calculated.

Volume III of the design guide presents a summary of mechanical disciplines
and skills needed by the Mechanical Project Engineer to properly evaluate

the structural competence of an equipment system. As a spin-off from this objective, the Packaging Designer and the Structural Analyst will also find useful material in Volume III. The chapters are organized as to broad content in three categories; analysis, validation, and hardware topics.
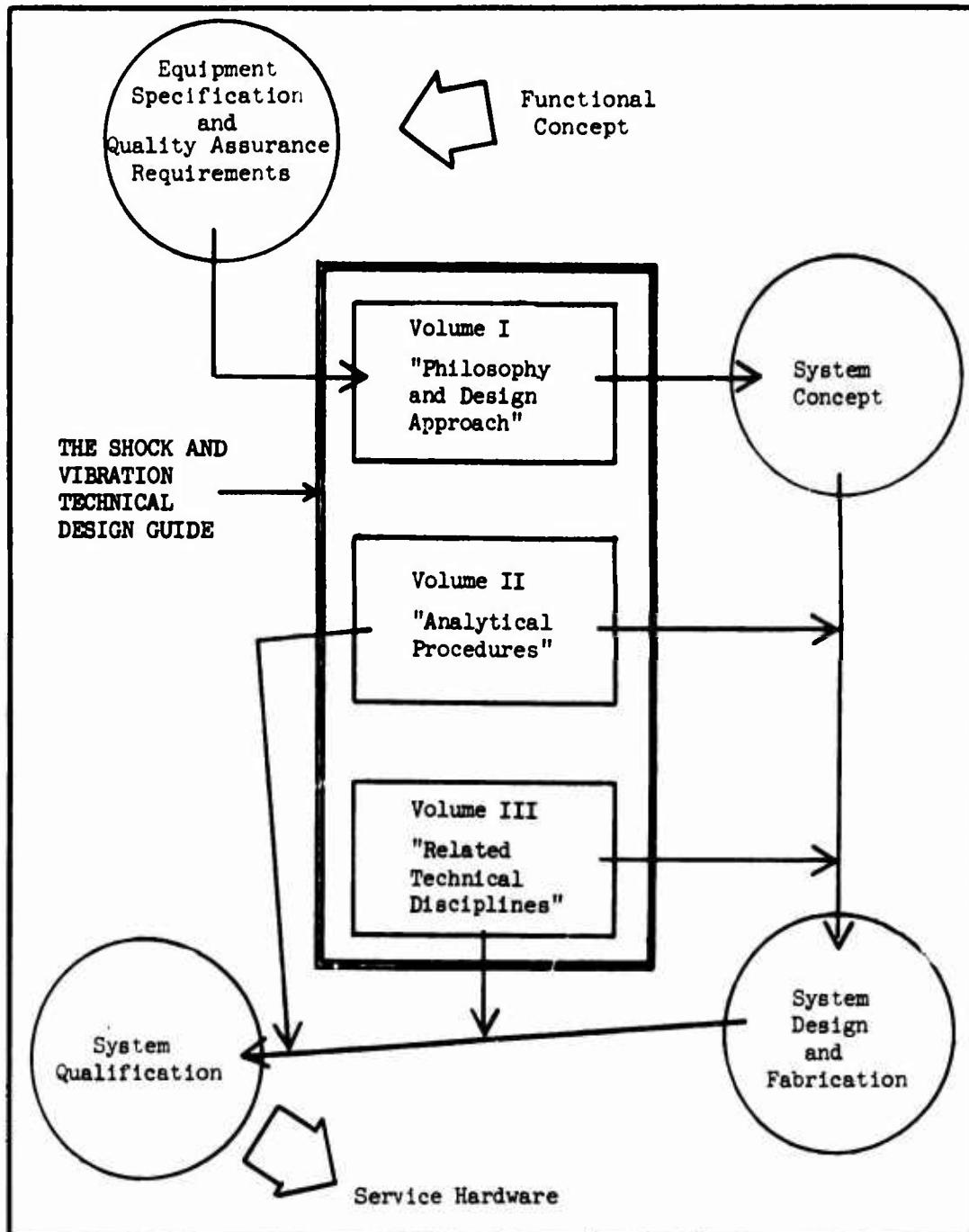
SUPPORTING EQUIPMENT STRUCTURAL INTEGRITY: The Design Guide supports the equipment development program from functional concept through service-ready hardware.

THE CHARACTERISTICS OF A VIBRATING SYSTEM

The long time repetitive aspects of vibration can cause serious fatigue problems in equipment structure, as well as component malfunctions.

Vibration, as the term is used in this Design Guide, is an oscillatory motion of a mechanical system. Vibration is characterized by a mean or central value about which the oscillation occurs, an amplitude describing the peak intensity of the vibration, and a frequency which describes the rate of occurrence of the excitation.

Service environment vibration is often found to be a repetitive, sinusoidal excitation, a situation which is called steady-state. Occasionally, vibratory components occur as a super-position of many sinusoidal quantities, each of which are integer multiples of some fundamental frequency. The vibration characteristic then repeats itself after some finite time interval; the resulting excitation is known as periodic. If no discernible pattern or repetition exists among the vibrational components, then the excitation may be described as complex.

Vibration may be further described in terms of the method of excitation. If the disturbing force is periodic or repetitive, then the resulting excitation is steady-state and deterministic; that is, the value of some future vibratory excursion may be predicted from the pattern of the recorded vibration-time history. In contrast, if the vibratory excitation is completely unpredictable, then the response is known as random.

The vibrating system may be either forced or free. A free vibrating element continues to oscillate after being disturbed without external interference, until the excitation dies out due to damping. A forced vibrating system is excited by some driving disturbance, an excitation which may be either random or deterministic, depending upon the characteristic of the source. Free vibration often results from a sudden pulse of energy, or shock, and is then transient in nature. The frequency of the oscillation will be the natural or fundamental frequency of the structural system. The response frequencies of a forced vibrating system will contain elements of the forcing frequency as well as response frequencies of the structural elements.

Structural failure resulting from vibrational excitations usually takes the form of fatigue or progressive fracture of the material emanating from an incipient crack or blemish. Fatigue strength of engineering materials varies with the number of iterations of load, as well as the stress intensity. Thus, it is important to consider the frequency content of the load as well as the load intensity. The load intensity in a dynamic system is greatly influenced by the resonant characteristics of the oscillating element.

Resonance in a mechanical system causes the maximum excursion of the system for a given input. Resonance exists when a minute change in frequency of the stimulus causes a decrease in the system response. The value of this frequency is known as the resonant or natural frequency. The lowest numerical value of resonant frequency is the fundamental.

The level of the resonant response, or resonant rise, is a function of the damping characteristics of the dynamic system. The lower the damping quotient, the higher the resonant response of the system (or quality factor Q) and the sharper the peak. Highly damped systems exhibit a broad response near the resonant frequency. Damping is the dissipation of energy in a dynamic system which manifests as a diminishing of system response with time. This basic concept is illustrated below.

VIBRATION CHARACTERISTICS

Response

Steady State, Sinusoidal Excitations

Multiple, Periodic Excitations

Complex, Random Excitations

Time →

RESONANT CHARACTERISTICS

Little Dampening (Q High)

Highly Dampened (Q Low)

Amplification

Transmissibility Ratio

1.0

Natural Freq.

Attenuation

Frequency →
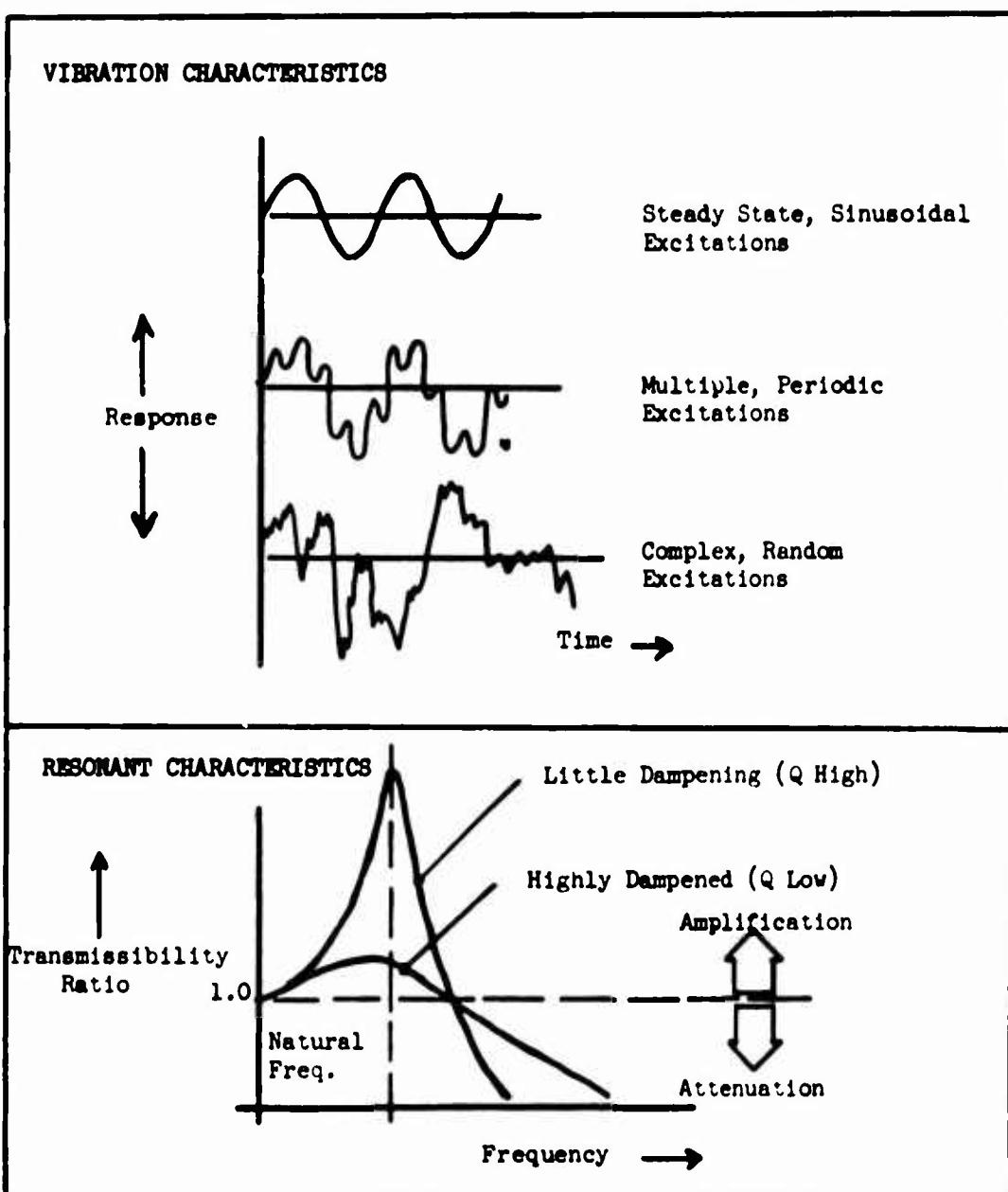
THE VIBRATING SYSTEM: A system excited by a vibrating impulse has response characteristics which contain elements of the disturbance as well as the system itself.

SHOCK EFFECTS IN EQUIPMENT STRUCTURE

Shock may cause initial fracture, overexcursion, or functional failure in electronic
equipment. Fatigue damage is also a potential hazard.

Shock in a mechanical system may be described as a sudden transient dis-
placement, a sudden change in velocity, or an acceleration experienced
over a relatively short period of time. Shock excitation is non-periodic,
and is described in terms of a pulse, a step or a transient load occur-
rence. Shock is always characterized by the concepts of suddenness and
severity.

The effect of shock stimuli on equipment structure is similar in some
respects to the effect of vibrational excitation. If the shock pulse is
short in duration, then the structural response will reflect the natural
frequency of the structural element, with acceleration intensities pro-
portional to the severity of the impact. If the shock pulse is relatively
long or has distinct frequency characteristics, then the resulting
response will often reflect both the period of the impulse and the natural
resonances of the responding system.

Shock occurs in the service environment of Army electronic equipment
packages primarily during transport and handling. An exception is the
effect of ballistic impact resulting from a near-miss explosion or shell
impact near the equipment. A range of Quality Assurance Tests have been
devised to simulate this environment, and are discussed in detail in later
sections.

Shock inputs to Army equipment resulting from test requirements are nor-
mally specified either by pulse characteristics, as in the case of air-
borne equipment, or by test parameters, as in the ballistic impact test.
The pulse characteristics specified are pulse shape, peak acceleration,
and pulse time duration. The test parameters most often used are hammer
height before impact or specimen drop height, as typical examples.

Both types of shock specification may be illustrated for the designer in
terms of shock spectra. Shock spectra is a plot of static equivalent
acceleration vs natural frequency, and will be used throughout this
Design Guide to describe the shock motion. The shock spectra may be
thought to be the response of a series of single degree-of-freedom mass-
spring systems to the imposed impact load. This effect is plotted in
terms of the natural frequency of each of these spring-mass complexes,
and thus is analogous to structural elements of the same resonant fre-
quency. These shock spectrum plots in the frequency domain present the
designer with a convenient tool for analysing the equipment structural
response characteristics to a given shock stimulus, based upon analogy
with a simple single degree-of-freedom system.

Failure in an equipment package due to shock stimulus may occur in various
ways. A functional anomaly resulting from the impact, such as relay
chatter or switch drop-out, is a common occurrence in shock testing.
Material fracture or excessive plastic deformation are other failure modes
which may result from the first acceleration of the equipment system.
Excessive excursion of the unit as a whole is also a common impact
failure; this motion response may not necessarily be disastrous by itself,

but may cause secondary collision impacts or bottoming of isolators which
in turn may cause structural failure. A series of transient shocks occur-
ring at regular intervals may excite a structural element into a resonant
vibration with subsequent fatigue failure; an example is the bouncing
excitation experienced during transport over rough terrain. Any of the
above failure modes may also cause an operational malfunction, a fact
which must be considered when designing support structure and evaluating
component fragility.

ACCELERATION, VELOCITY, AND
DISPLACEMENT CHARACTERISTICS
OF A HALF-SINE SHOCK PULSE

SHOCK SPECTRUM TRANSFORM OF
A HALF-SINE SHOCK PULSE
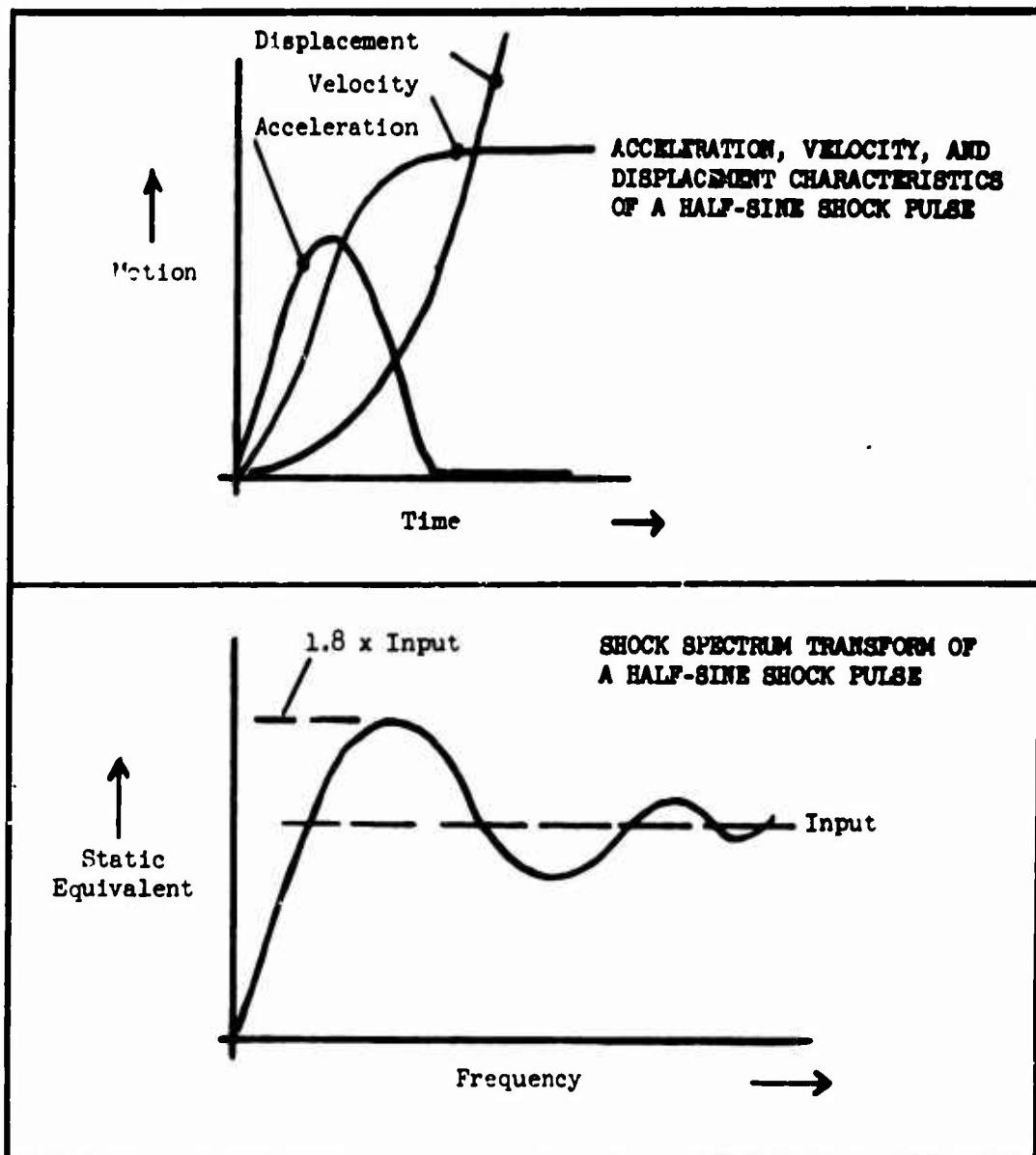
SHOCK RESPONSE: The half-sine shock pulse may be transformed into a
workable shock spectrum.

THE SINGLE DEGREE-OF-FREEDOM CONCEPT

The single degree-of-freedom idealization aids in simplifying the complex calculations associated with dynamic analysis.

The single degree-of-freedom approximation of a complex mechanical system is used to simplify the analytical techniques needed to evaluate an equipment system. In an analytical area as complex as the determination of the shock and vibration responses of structurally complicated equipment packages, simplification is necessary.

The degree-of-freedom of a mechanical system is the number of independent coordinates required to define completely the position of all parts of the system at any instant of time.[5] This concept is manifest as the number of separate, independent displacements that are possible in the system's motion. The single degree-of-freedom system then, is one for which only one orthogonal coordinate is necessary to completely define the position of the system at any instant in time when the system is displaced as a result of a dynamic forcing function.

This approach will yield the fundamental resonant response of the equipment system, an approximateion that will give valid results in most cases. An example is an equipment element enclosed in a relatively stiff cabinet, and mounted on relatively soft isolation mounts. The single degree-of-freedom idealization very adequately describes the response characteristics of the package. The individual equipment components within that package are not treated analytically however, since more information is needed to describe the transfer characteristics of the dynamic excitation through the complex structure and thence to the component. This idealization is also consistent with the input stimuli resulting from the Quality Assurance provisions. For the most part, these tests are required for the entire system and not the individual components. The input parameters are normally specified at the interface of the test machine and the equipment mounting structure. Thus, the input requirements for Army equipment are, in a sense, described in single degree-of-freedom terms.

The response curve for the single degree-of-freedom model graphically describes the effect of damping on the resonant rise. If little damping is present in the system, then high peak response may be expected. The frequency range above the fundamental frequency however, exhibits a lesser response than the highly damped system. On this basis, it can be expected that equipment elements having natural frequencies above the fundamental of the system will experience a higher input to the element from the system when the system is highly damped. The extent of damping which is built into a system must be carefully selected, with consideration given to the effect of resonance on the high natural frequency components. The relationship existing between the parameters of resonant rise, natural frequency, and damping ratio are well illustrated with the single degree-of freedom response curve.

THE SINGLE-DEGREE-OF-FREEDOM MODEL

Mass (M)

Response

Stimuli

Spring
Rate

(k)

(ζ)Critical Damping Ratio
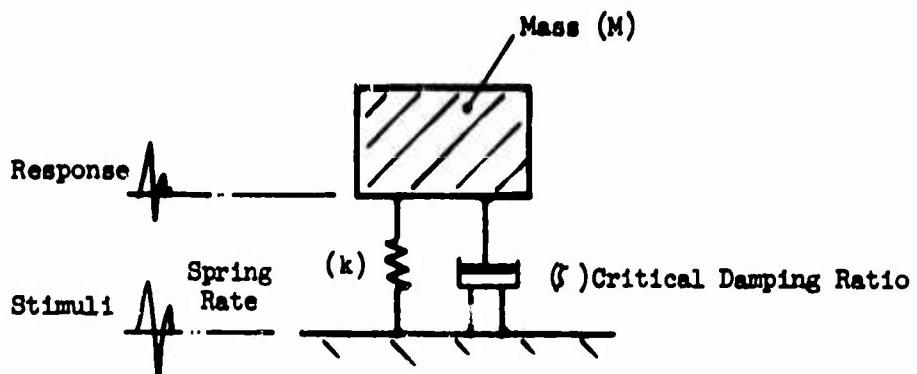
FREE OSCILLATION OF THE SINGLE-DEGREE-OF-FREEDOM SYSTEM

Displacement

Time →

Oscillation
Decay Due to Damping
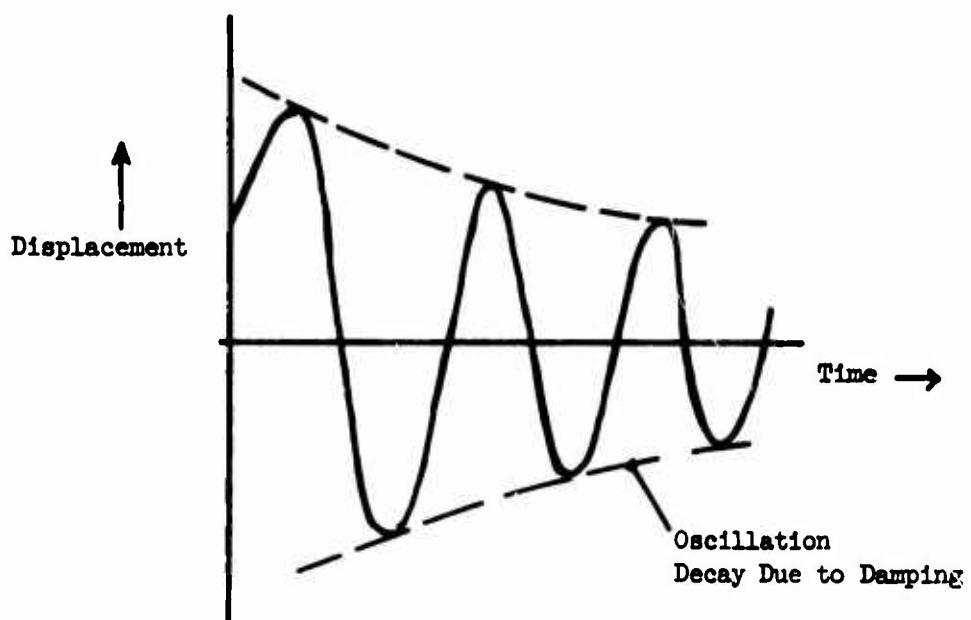
THE SINGLE-DEGREE-OF-FREEDOM CONCEPT: A convenient analytical method for
a complex system.

CONTROL OF THE DYNAMIC ENVIRONMENT

The severe effects of shock and vibration disturbances may be controlled by adjustment of structural response, or isolation, or elements of both techniques.

The control of the shock and vibration environment to Army electronic equipment systems is accomplished largely by the implementation of two procedures or their combination. These are; the isolation of the equipment from the dynamic environment and/or the reduction of the response characteristics within the equipment. The reduction of the dynamic stimuli at its source is a third alternative, which has only limited applications to Army equipment. The control of the source of dynamic excitations in order to reduce the severity of the equipment response involves the balance of moving parts and the improvement of fits and clearances. This alternative is not always open to the designer of Army equipment packages. The environment is usually either specified, or is beyond his control. However, when possible, reduction of the source of the load is an obvious solution to severe loading problems.

Isolating the equipment from dynamic stresses may also be accomplished by isolating the source of the disturbance. This approach might be helpful in the case of equipment which is required to operate while a transport vehicle is in motion. This method would also include the 'solation of offending machinery in the vicinity of sensitive equipment. Isolation of the equipment itself from the effects of the environment is the most often applied approach to packaging fragile electronics. When the dynamic stimuli is specified by a Quality Assurance provision, reduction of the excitation at the source is not feasible.

Isolation systems are discussed in more detail in the chapter on "Dynamic Attenuation" in Volume III of the Design Guide. There are elements of conflict in the practice of isolation techniques; the frequency and damping characteristics which offer the best protection from the vibration environment, are far from optimum for resistance to impact loads. Isolation systems which are designed to successfully pass Quality Assurance tests may sometimes fail in service due to the inability to completely duplicate the operational environment in the laboratory.

The most effective approach to the control of the dynamic environment lies in the assurance of the dynamic integrity of the equipment itself. If the structure is sufficiently strong to sustain the dynamic stimuli without loss of function, then the design is successful and the operational suitability of the system will be high. The most widely used design method for response reduction within the equipment structure is the improvement of the stiffness qualities of the structural elements. The structural factors that affect stiffness, (material elastic modulus, mass distribution, and support geometry) are the same factors that affect natural frequency. Thus an increase in stiffness denotes an accompanying increase in natural frequency.

In general, the natural frequency for sea transportable equipment should always exceed 35 Hz, since most of the shipboard excitations will be below this value.[1] If the normal static deflections of all the structural members due to their own 1g loads are less than about 0.008 in., then this approximate frequency limit will be met.

Dynamic energy may also be dissipated within the equipment structure by increased damping. This in turn reduces the amount of resonant rise at the critical frequency, without appreciably affecting the frequency itself. Energy may also be absorbed within auxiliary resonant masses, tuned to dissipate as much of the disturbing stimuli as possible.

ISOLATION

- Isolation of the entire system

- Isolation of the fragile elements

- Isolation of the source

- Reduction of stimuli at its source

RESPONSE

- Use of rugged components

- Improved stiffness

- Decreased mass

- Proper use of damping

- Application of tuned resonators

CONTROLLING THE ENVIRONMENT: The destructive effects of the imposed shock and vibration environment may be met in an equipment structure by the management of isolation or response, or some combinations of each.

VOLUME I

METHODOLOGY AND DESIGN PHILOSOPHY

SECTION 2 - STRUCTURAL DESIGN CRITERIA

- Transport Categories to Define Equipment Class

- Quality Assurance Provisions and Equipment Class

- Limitations Inherent in the Quality Assurance Tests

- Some Typical Army Transport Methods and Vehicles

TRANSPORT CATEGORIES TO DEFINE EQUIPMENT CLASS

The four transport categories of importance to equipment class definition are manpacked, vehicular mounted, shelter and van mounted, and airborne.

The Army Electronic Equipment Command designates service and Quality Assurance test requirements for their equipment by "equipment class". This class designation differentiates categories by the mode of equipment transport, and by the operational requirements of the equipment systems as mobilized. The equipment class also defines the loading environment in terms of the Quality Assurance tests normally required of equipment manufacturers. Thus the equipment class may be expressed in terms of an envelope of loading parameters reflecting the individual test inputs constrained to the class. There are exceptions to the class criteria in the case of special equipments that fit none of the categories; these requirements will then normally be detailed in the equipment specification.

There are four equipment mounting categories which are of importance in the definition of equipment class. These groups are manpack, vehicular mounted, shelter and van mounted, and airborne.

Manpack equipment is generally operated in the field without benefit of vehicle, shelter, or van protection. The equipment is constrained to provide its own environmental protection in service, a factor which might have impact on dynamic structural integrity. These equipments are highly portable and are often housed in a combination transit case and environmental cover. Equipment in this category is normally transported loose on the cargo area of the transport vehicle, and is always operated externally from the vehicle.

Vehicular mounted equipment is usually firmly affixed to the transport vehicle, either with or without isolators. The equipment is thus constrained to operate within the vehicle; the equipment function may occur with the vehicle moving or stationary, as dictated by the system specifications. Tht vehicle may be expected to provide a measure of environmental protection, and will also attenuate some of the dynamic excitations emanating from the terrain. It is also possible for the vehicle to magnify certain frequencies, a factor which must be carefully considered. Both tracked and wheeled vehicles are included in this category; the resulting excitations differ greatly in their input to the equipment. A summary of the road mobility environment is given in the testing chapter, Volume III, and is also imposed by equipment class loading criteria presented in Volume II.

Shelter or van operated equipment is normally mounted and operated from within an enclosure, and thus receives the full benefit of the protection provided by the shelter. Shelters in this category are usually transported on the cargo bed of a transport vehicle, or are mobilized by wheel complexes where the shelter itself serves some of the chassis function.

Airborne equipment may be either transported by aircraft, or may be constrained to operate within the aircraft while the vehicle is in flight. Virtually all equipment must be capable of air shipment as a means of transport, while only class 6 equipment is required to function in flight. The details of these requirements are delineated in paragraphs 3.0 and

4.0 of the equipment procurement specification. Any special requirements not covered by equipment class or transport category will also appear in these paragraphs.
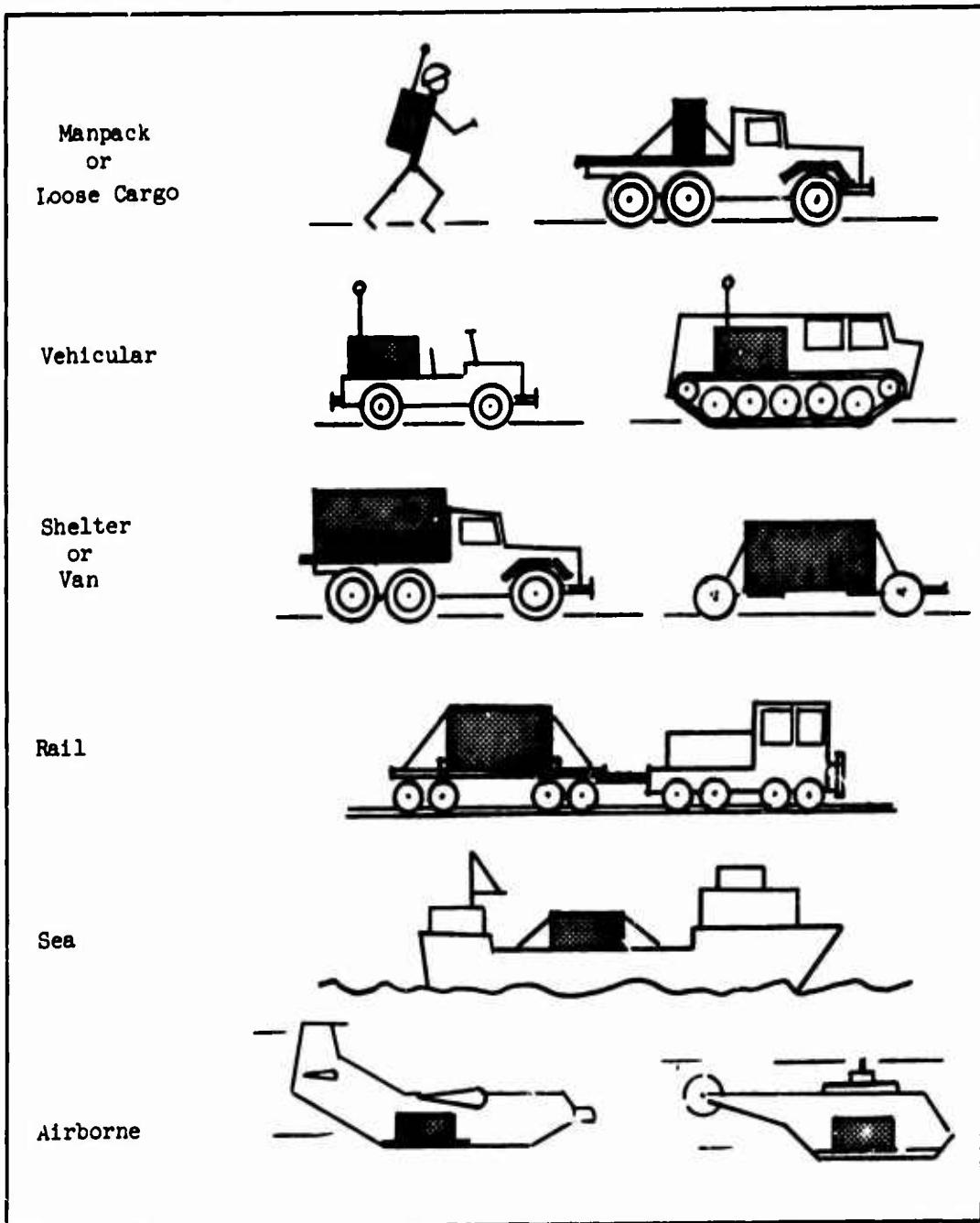
EQUIPMENT CLASS AND TRANSPORT MODE: Transport categories for Army equipment reflect the mounting method, and mobility mode.

QUALITY ASSURANCE PROVISIONS AND EQUIPMENT CLASS

The equipment class designation used by the Army in procurement specifications is a convenient method of summarizing the required Quality Assurance tests.

The transport mode and operational requirements are generally designated by the applicable equipment class in Army electronic equipment procurement specifications. In addition, the equipment class also delineates the required Quality Assurance provisions that the equipment systems must pass to be approved by the contracting officer. This class categorization may then be used to define the shock and vibration excitations resulting from the required tests, an approach which is employed in Volume II of the Design Guide to develop input load criteria. The adjacent figure summarizes the required shock and vibration tests according to equipment class.

The tests are also delineated for the type of excitation that is input to the system; i.e., steady-state vibration, random vibration, or shock. The steady-state or sinusoidal vibration excitations are well defined as peak acceleration vs frequency plots since acceleration, velocity, displacement, and frequency are definable by dynamic relationships. Random excitations will be presented as power spectral density plots (G^2/Hz vs frequency). Statistics may then be employed to express the probability of experiencing a given fraction of peak acceleration for a given frequency interval. The shock excitations are presented in this guide as shock spectra plots, or static equivalent acceleration vs frequency spectra. The assumption is that any system with a given natural frequency will respond in the same manner to a given stimulus. Thus, the shock spectra may be related to the structural system under analysis for a specific natural frequency.

The matrix of tests presented in the adjacent figure is typical for the usual equipment systems encountered in Army electronic equipment procurement specifications. There are exceptions however, and the equipment specification must be reviewed for the exact requirements. The failure and acceptance criteria will also be defined in the equipment specification.

The resonant search vibration requirement and loose cargo bounce tests for Class I equipments are designated for special cases only. If a given dynamic test is specified for equipment in the operational mode, then the same test is not normally repeated for the same equipment, non-operating.

Classes of Equipment: Army equipment systems shall be considered to be of the following classes, according to their basic use and the mode of transportation employed in that use:

Class I: Includes equipment which will be field transported as loose cargo in vehicles and/or manpack. Equipment designed in a combination shipping and operation container or transit case is included in this class. The equipment shall be capable of being shipped by rail, truck, sea or air.

Class II: Includes equipment which is installed in (firmly affixed to) an unarmored vehicle, shelter or van and is operated when the vehicle, shelter or van is not in motion. The installed equipment shall be capable of being shipped by rail, truck, sea or air.

Class III: Includes equipment installed the same way as in Class II and is operated while the vehicle, shelter or van is in motion.

Class IV: Includes equipment which is installed in (firmly affixed to) a tracked vehicle, or shelter, or van mounted to a tracked vehicle, and is operated when the vehicle is not in motion. This vehicle may be either armored or unarmored. The installed equipment shall be capable of being shipped by rail, truck, sea or air.

Class V: Includes equipment installed the same way and in the same vehicles as in Class IV but may be operated while the vehicle is in motion.

Class VI: Includes equipment that is installed or transported by aircraft. The equipment may be either operated or stowed while the aircraft is in flight.

Test To Be Performed	Equipment Class					
	I	II	III	IV	V	VI
Equipment Operating						
Vibration (Resonance Search)(10-55 Hz)	X					
Vibration (5-500 Hz)						X
Vibration (Resonance Dwell)						X
Bounce, Loose Cargo	X					
Bounce, Vehicular			X		X	
Shock, Ballistic					X	
Munson Road Course			X		X	
Perrimen Road Course			X		X	
Equipment Non-Operating						
Vibration (Resonance Search)(10-55 Hz)	X	X		X		
Bounce, Loose Cargo	X					
Bounce, Vehicular		X		X		
Shock, Ballistic				X		
Shock, Bench Handling	X	X	X	X	X	
Shock, Drop	X	X	X	X	X	
Shock, Crash Safety						X
Munson Road Course		X		X		
Perrimen Road Course		X		X		
Railroad Hump Test		X		X		

LIMITATIONS INHERENT IN THE QUALITY ASSURANCE TESTS

The Quality Assurance tests are representations of the service environment that may be encountered by Army equipments. Exceptions and limitations may also constitute a design loads constraint.

The function of the required shock and vibration Quality Assurance tests is to guarantee a minimum standard of structural integrity in the equipment package, which in turn reflects a confidence that the equipment will survive the service environment, and function as intended in the field. This intention therefore assumes a certain correlation between the service environment and the contractural tests that the equipment system is required to pass. In general, the shock and vibration tests required of Army equipment packages are based on the upper average of the dynamic excitations the package will experience in service. An equipment element that successfully passes the required tests will probably survive the average environment encountered in service. However, in some cases, an equipment system that passes all tests successfully may still experience some difficulty with the environmental extremes. The Quality Assurance tests serve as a basis for equipment reliability, and are not absolute quarantees of dynamic integrity.

The design implication here is clear. The equipment system must pass the required tests and the equipment must also function in the field. The responsible equipment engineer must be aware of both constraints and must evaluate the input excitations resulting from both experiences, and assess their influence on his particular equipment system.

Looking at the contractural Quality Assurance tests in terms of the type of excitation imparted to the equipment, the following limitations are apparent:

1. Vibration - The resonant survey for Army systems begins at 10 Hz for ground equipment, and 5 Hz for airborne equipment. This lower cutoff is for the convenience of the test machines which are sometimes amplitude limited in the lower frequency ranges. The actual service environment has inputs down to nearly zero Hz. This is apparent from the low speed capabilities of trucks, ships, helicopters, and other transport vehicles. Thus, an isolation system or structure that has resonant frequencies below about 10 Hz may protect the equipment during qualification testing, but may also be a major contributing factor to failure in transport.

2. Shock - One of the most severe shock tests is the railroad humping impact, a coupling collision tested at 7.0 mph. Rail practice surveys reveal that coupling impacts may occur at speeds up to 20 mph. Since the shock intensity is proportional to the rail car impact speed, there is a degree of risk involved in designs which meet the contractural Quality Assurance test levels marginally.

    Similarly, the shock intensities experienced during handling drop tests are proportional to the drop height. It is reasonable to conceive of rough handling situations of shipping packages in the field which would exceed these limits.

Ballistic shock test inputs to the equipment package are dependent to a degree on the characteristics of the test specimen. Since the test machine does not have infinite impedance, the specimen may feed-back energy as a two degree-of-freedom system, and thus influence the impact intensity and frequency content at the machine-specimen interface.

3. Random Excitations - This test category which includes cargo bounce and vehicular bounce tests, is quite representative of the actual shipping environment. The same problem of specimen-machine feedback exists in these tests. The random aspect of the test also contributes to weakness in input load definition.
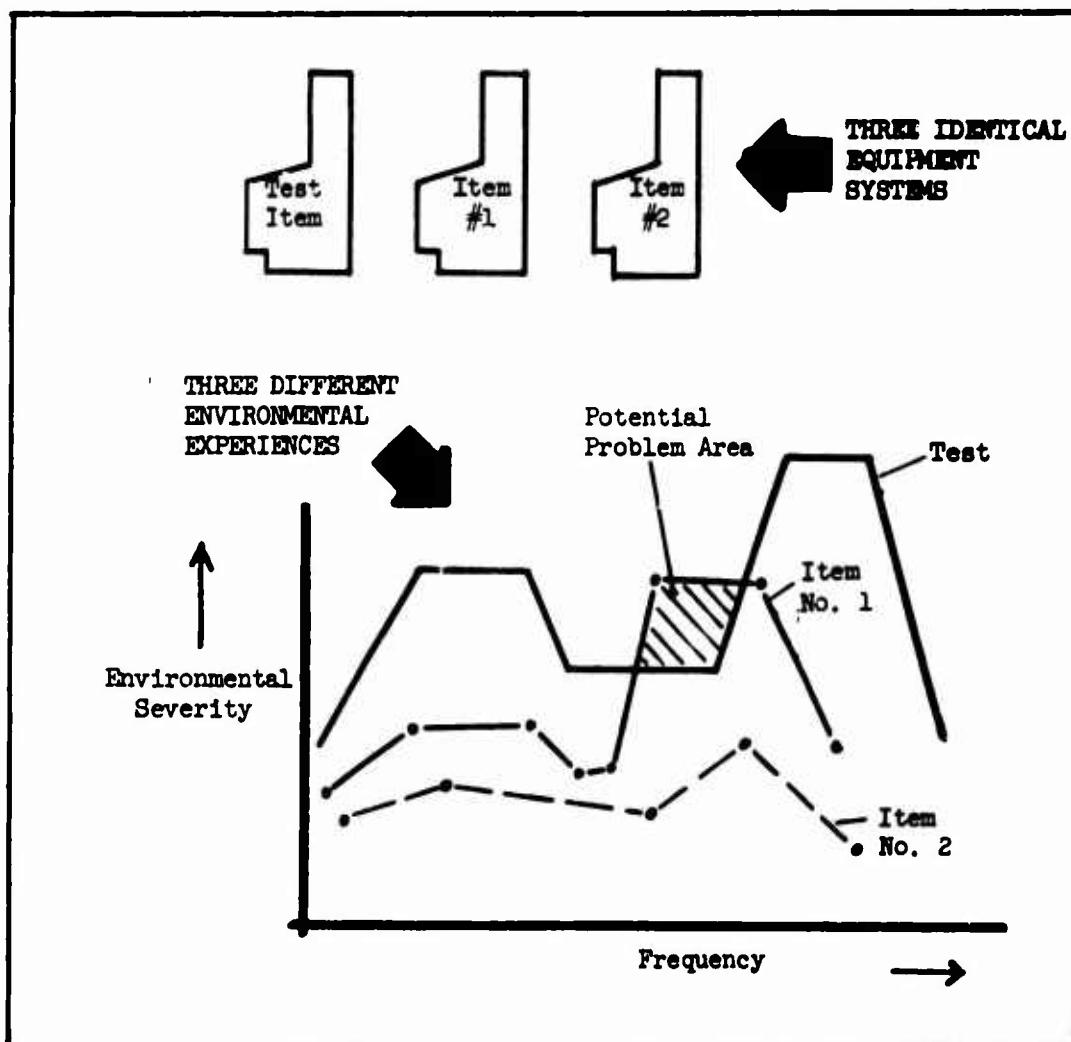
LIMITATIONS: Quality Assurance tests are representative of an average severe service environment. It is possible for an individual equipment to fail in service in spite of a successful test qualification.

SOME TYPICAL ARMY TRANSPORT METHODS AND VEHICLES

Two categories of vehicles are important to the Army equipment classes; those vehicles to which the equipment is firmly affixed, and those vehicles which transport the equipment as loose cargo or tied-down modules.

VEHICLES USED TO FIELD-TRANSPORT LOOSE CARGO OR EQUIPMENT MODULES
(The Equipment is Usually Non-Operating)

Vehicle (Example)	Environmental Characteristics
Rail	
Flatcar	Low and Intermediate Frequency,* Periodic Vibration
Boxcar	High Impact Shock During Coupling (Vertical and Longitudinal)
Truck-Cargo	
Wheeled Vehicles (M35)	Low Impact Bounce, Low and Intermediate Frequency Random Vibration
Tracked Vehicles (T 6)	Low Impact Bounce, Intermediate Frequency Periodic Vibration*
Sea	
Cargo Ship (AKA)	Low Frequency Vibration, High Excursion Inclination
Amphibious Carrier	Low Frequency Vibration, Inclination, and Moderate Bounce
Air	
Jet Cargo Aircraft (C-141)	High Frequency Vibration,* Crash Safety Shock
Propellor Cargo Aircraft (C-130)	Intermediate Frequency Vibration, Safety Shock Crash
Cargo Helicopter (CH-47)	Low Frequency Vibration, Crash Safety Shock

VEHICLES IN WHICH EQUIPMENT MAY BE INSTALLED
(Equipment May Also be Operating in Motion)

Vehicle (Example)	Environmental Characteristics
Tracked	
APC (T116)	High Impact Ballistic Shock, Short Pulse Duration
Tank (M-60)	Random Vibration and Intermediate Frequency Periodic Vibration*
Wheeled	
Shelter Module Mobilizer (XM-720) Transported by Truck (M35)	Low Frequency Periodic Vibration*
Van (Trailer - M359) (M220)	Random Vibration
Jeep (M38)	Moderate Shock and Moderate Pulse Duration
Airborne	
Helicopter (CH-47 Chinook)	Low Frequency Periodic Vibration*
Propellor Aircraft (OV-1C Mohawk)	Intermediate Frequency Periodic Vibration* and Crash Safety Shock

* NOTE: Approximate frequency limits given qualitatively, correspond to the following ranges:

Low Frequency	-	0-55 Hz
Intermediate Frequency	-	30-500 Hz
High Frequency	-	200-2000 Hz

These frequency ranges are by no means exhaustive, but represent the average experiences which may be encountered.

VOLUME I

METHODOLOGY AND DESIGN PHILOSOPHY

SECTION 3 - DESIGN APPROACH TO STRUCTURAL RELIABILITY

- Some Ground Rules for Successful Design of Equipment Structure

- Countering Shock and Vibration Effects in Army Equipment

- Designing Equipment Structure for Stiffness and Lightness

- Tailoring the Natural Frequency Parameter to Suit the Environment

- The Application of Damping to a Structural System

- Mounting Fragile Components in Equipment Packages

- A Design Outline for Improved Structural Dynamic Integrity

SOME GROUND RULES FOR SUCCESSFUL DESIGN OF EQUIPMENT STRUCTURE

Some of the most important preliminary considerations in the design and analysis of equipment structure are often the ones that are overlooked until very late in the program.

After the elements of terminology and the analytical philosophy of dynamics are made familiar to the equipment designer, he must then address himself to the problem of; "Exactly how do I proceed into the design of support structure for my equipment package?" The first steps in this process are often the most elementary and unfortunately are also the ones most often overlooked. The art and science of designing for structural reliability is of such inherent complexity that certain fundamental considerations sometimes are lost in the static of the signal.

It is basic that the designer must make himself thoroughly familiar with the procurement specification, particularily those sections that deal with the intended service environment and the required Quality Assurance provisions. Often, the very rigorous shock and vibration test provisions contracturally required of the equipment system are not fully evaluated until the equipment is designed, fabricated, and ready for test. It is economically too late to disciver that the system must be subjected to a ballistic shock test, for example, when the pre-production system models are ready for validation. Panic fixes that are generated at this point in time are usually not very sound technically, and are invariably expensive. They also tend to compromise the intended system function.

Rule No. 1: Read the equipment procurement specification - understand the imposed test and service environments - relate these requirements to equipment class, including any special test provisions.

Once the exact test requirements are firmly established, it remains to relate these requirements with resulting structural excitations. The analytical procedures outlined in Volume II are based upon three classes of excitation; sinusoidal vibration, shock, and random vibrations. Analytical procedures are proposed for each of these situations. It follows then, that the designer must plot an envelope of his particular test environments. The detail data corresponding to each of the test requirements is available by equipment class in Volume II, and by individual test category in the chapter on testing in Volume III.

Rule No. 2: Plot the similar excitations resulting from Quality Assurance test provisions on the same scale in the frequency domain - pick out the critical frequencies and accelerations - assess the fatigue damage potential by establishing the number of iterations of load resulting from each of the excitation elements.

The shock and vibration excitations inherent in the standard dynamic tests are sometimes truncated to suit the test machine and procedures. These limitations are discussed in detail in the chapter on dynamic simulation, Volume III. It is a fact of life that tests only approximate the real world in some cases, and the service environment may differ substantially from the imposed test requirements. The designer must be constantly aware of this limitation and cover these areas in his design. An example of this difference is the lower limit of the vibration tests, 10 Hz. Many excitations in service will dwell for extended periods of

time at frequencies in the 0-10 Hz range. The test cutoff is for the convenience of the test machine only. Soft isolation systems in this range could cause serious field failure due to low frequency resonance, if this possibility is not considered in the design analysis.

Rule No. 3: Understand the service environment needs- recognize the possibility of some differences between the imposed test excitations and the real world in the field.

There are serious differences in the response of structural systems to shock vs vibration excitations. The following topics in this Volume deal with the details of these divergent design approaches. Shock pulses of short duration for example, will usually excite the higher frequencies in a structure. Vibration associated with the road mobility environments will cause excitation of the systems to nearly zero Hz. Soft mounting systems may survive shock loads very nicely, but may also cause extreme resonant behavior from a low level vibrational disturbance. Stiff structure may survive the low frequency vibration environment very nicely, but may precipitate failure of secondary components due to high transmissibility of only minor shock loads. It is imperative to recognize the basic disparity in design philosophy for shock vs vibration; an equipment structure must be analyzed with both environments in mind. Design adequacy for shock loads does not insure equal structural integrity for vibrational disturbances.

Rule No. 4: Consider the combined effects of both shock and vibration in the analysis of equipment structure - note the conflict between the two types of dynamic experiences - all equipments are subject to elements of both excitations.

RULE NO. 1 — Read and understand the procurement specification.

RULE NO. 2 — Plot all of the dynamic excitations on the same piece of paper, to the same scale.

RULE NO. 3 — Understand the service environment and the impact of potential differences with the Quality Assurance test provisions.

RULE NO. 4 — Consider the effects of both shock and vibration, and recognize the conflicting needs of the two excitation types.

BASIC GROUND RULES: There are certain fundamental steps that the designer must follow before the design analysis is firm. These preliminary steps are elementary and often overlooked.

COUNTERING SHOCK AND VIBRATION EFFECTS IN ARMY EQUIPMENT

The design approaches for countering dynamic excitation are twofold; live with it or keep it out. Coupled with this choice are the divergent response characteristics of soft vs stiff structure.

There are two basic approaches of importance to the equipment designer for countering the effects of shock and vibration excitation in Army equipment; provide support structure and component capability of sufficient integrity to withstand the raw environment without failure; or insulate all or part of the environment away from the fragile elements. Since the equipment designer has little control over the intensity of raw environment, then these two steps, or combinations of them, are the primary alternatives open to him.

Equally dichotomous are the diverse effects of shock and vibration on structure. Shock which is characterized by accelerations for relatively short pulse durations, requires large elastic excursion capability to dissipate the impact energy without bottoming or over-excursion. The lower frequency ranges (or "soft" structure) may offer some escape since the shock environment is generally lower at the low frequencies, a fact that is apparent from the input excitations shown in Volume II, Section 2. The danger of over-excursion during shock is met by the use of stiff structure which experiences only small deformations from the impact. These two shock response effects are self-cancelling, a situation that creates a design dilemma.

Classically, vibration mounts have been soft devices implying low natural frequency. Unfortunately, the vibration environment extends to virtually zero frequencies. Thus, the soft mounts and soft structure will resonate within the service vibration frequency range contrained to the equipment, causing maximum excursion and maximum dynamic stress. This characteristic is at odds with the usual structural acceptance criteria of stiffness.

Two factors emerge from this apparent conflict between soft and stiff structure:

1. There is no pat design approach that is successful in all cases. Each structure must be evaluated on its own merits of stiffness and fragility just as each equipment class is subjected to a differing range of environments. What works for one may be disastrous for another.

2. Shock and vibration effects must be evaluated simultaneously. Minor changes in structural characteristics may cause little response change due to shock, but may cause failure from resonant vibration response, or vice-versa.

The characteristics of soft (low natural frequency) structure may be summarized as follows: soft structure may cause compound resonance of secondary components, wire leads, and shafting, which are usually low frequency elements; can cause secondary rocking modes of response; requires more space for excursion, larger packages, more danger from bottoming; best suited for fragile components within the equipment package; attenuates those excitations of frequencies above about 1.4 times the natural frequency of the element.

The higher natural frequency (stiff) structure also exhibits character-
istic response effects, such as: greater transmissibility of energy to
secondary components; less strain and excursion, which implies more com-
pact equipment packages; exhibits no energy attenuation capability below
the natural frequency of the basic ͟structure; generally causes less
fatigue of wires and small elements; limits the excursion due to over-
turning moments associated with high C.G. packages.
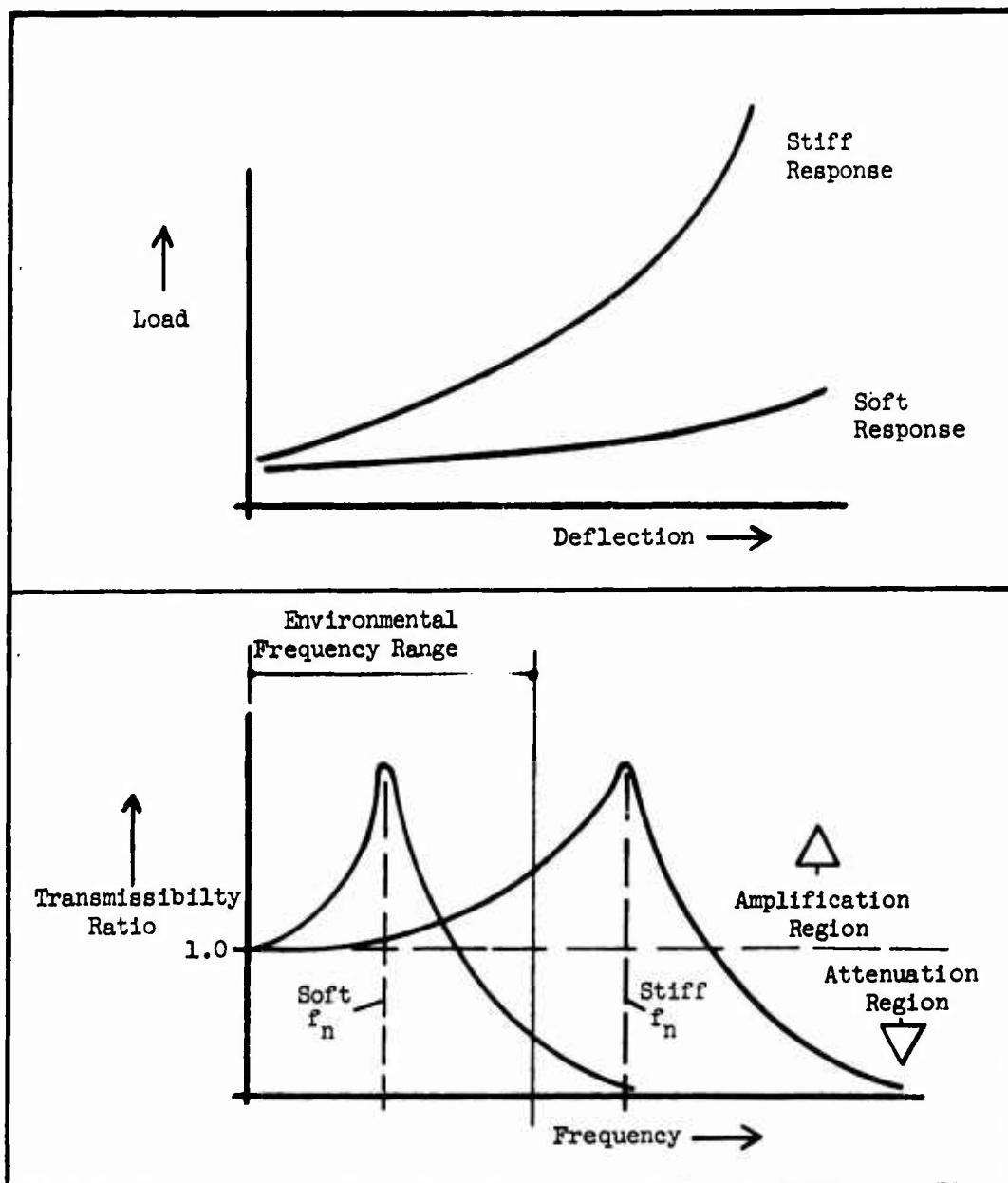
"STIFF" VERSUS "SOFT" STRUCTURE: The load response characteristics and
resonant response effects of structure vary markedly with stiffness or
natural frequency.

DESIGNING EQUIPMENT STRUCTURE FOR STIFFNESS AND LIGHTNESS

Stiffness and lightness imply higher natural frequency in equipment structure. The results are not always beneficial, however.

In the preceding topic, some of the tradeoff considerations concerning soft vs stiff structures were outlined. In general, stiffness is thought to be a desirable characteristic in equipment structure, although there are structural situations where the opposite is true.

Stiffness is a structural property characterized by small elastic deformations for a given load. The factors that tend to raise stiffness include high material elastic modulus, good creep resistance (to minimize time-load effects), high yield strength (to preclude plastic deformation), short bending spans, and high area moment of inertia in the structural sections. Stiffness in an equipment package does two things for the designer; stiff structures exhibit relatively higher natural frequencies, and stiff structures deflect less under a given load, thus reducing relative motion and required clearances, which is manifest as more compact packages. Stiffness in an equipment package also implies a greater ability to transmit energy through structure, thus causing a potential hazard to secondary elements that are mounted to the basic structure.

The classic transmissibility curve for a single degree-of-freedom system illustrates the attenuation problem in stiff systems. The excitations at frequencies below the fundamental are amplified at least one times the input; attenuation occurs only for frequencies above about 1.4 times the fundamental. Thus, if the fundamental resonant frequency of the package is higher than the environment, there will be no attenuation of energy from the support structure; the secondary components will feel the full intensity of the environment. This situation is not always fatal, but should be recognized by the designer.

There are several practical steps that the designer may take to improve the stiffness of the equipment structure:

1. Reduce the unsupported length of loaded members.

2. Provide moment carrying capability at beam support joints and structural interfaces.

3. Use structural members in tension and compression, and avoid bending elements, particularily the cantilever.

4. Use fasteners in shear. Avoid secondary bending and combined tension-shear interaction.

5. Use reinforcing elements (such as gussets) at joints and interfaces.

6. Use structural configurations that have inherent stiffness, such as the triangle and the truss. Avoid shapes that parallelogram under load.

7. Think in terms of capture rather than support when attaching elements to basic structure.

8. Use small members and miniature components where practical, as they tend to be more reugged and exhibit generally high natural frequency.

Of equal importance to the concept of higher natural frequency in structure is the avoidance of excess weight. Natural frequency varies directly with stiffness and indirectly with weight. Thus, a savings in one at the expense of the other can be self-cancelling.

Weight-saving techniques are generally familiar to the designer. Some ideas that may help reduce the weight penalty include:

1. Avoid unnecessary joints and interfaces.

2. Use the high strength-weight materials where feasible.

3. Use the light alloys where the design criteria is merely bulk.

4. Avoid redundant elements. Design for direct load paths that are easy to analyze.

5. Shave off excess material. Use lightening holes and truss-like configurations wherever possible.
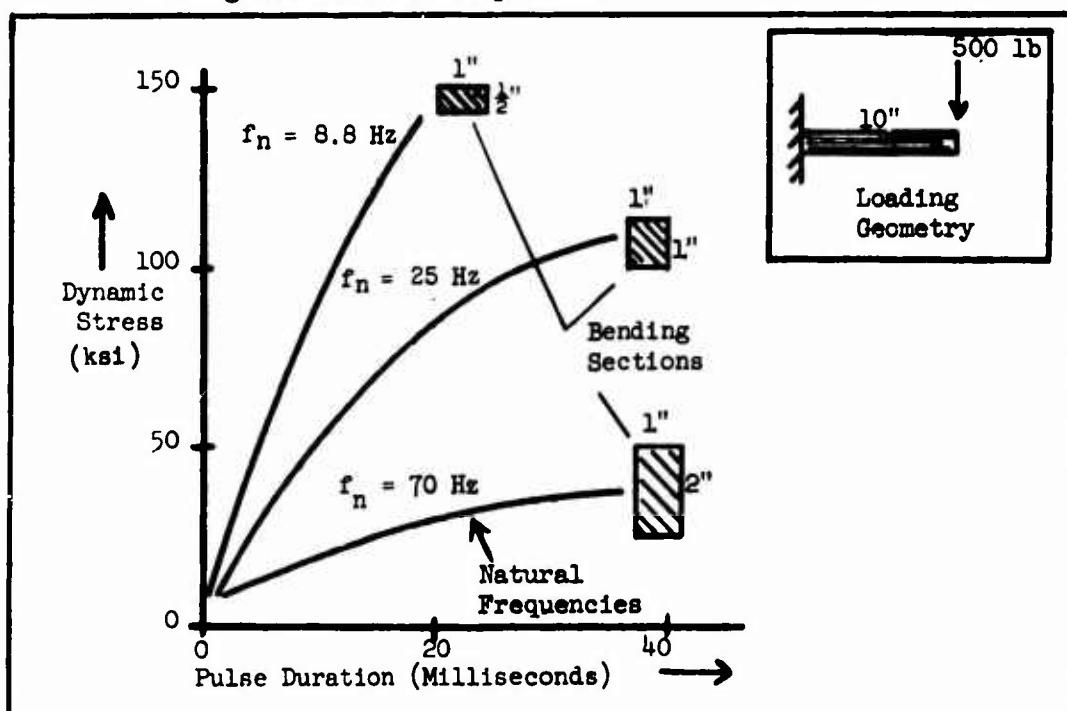
STIFFNESS EFFECTS: Stiffness may be improved by judicious use of beam section properties (1)

TAILORING THE NATURAL FREQUENCY PARAMETER TO SUIT THE ENVIRONMENT

The designer should study the frequency plot of his required shock and vibration inputs to optimize the fundamental frequency of his equipment structure.

The preceding topics have discussed the concepts of stiffness and lightness, and developed some design approaches to enhance these structural factors. The effect of changes in these parameters have a parallel effect on the resonant frequency of the structure, some of the details of which are discussed in depth in Volume III of this Design Guide. To reduce the design choices to the fundamental, there are basically two dynamic factors which the designer may adjust by changing the characteristics of the support structure; the natural frequency of the structure, and the degree of damping. Both factors affect the response of the equipment package to the imposed environment; the frequency parameter affects the frequency range at which the interaction occurs.

Natural frequency in simple structure may be estimated by calculating the static deflection of the element under one-g loading. Thus, all the structural factors which affect deflection also affect natural frequency.

To use the selection of the basic frequency range to advantage, the designer must first have a frequency-domain plot of the expected input excitations. To illustrate this approach, the figure at right is offered; a converted plot of random vibration response vs frequency for the road mobility experiences of the Munson Road Test. This particular plot is typical for wheeled vehicles, over all the Munson Courses, for a variety of vehicular speeds. This test requirement is often imposed on equipment modules that will be transported in the field by wheeled vehicle, such as the M-35 truck.

There is a definite lull in the resonant response curve for structure with a fundamental frequency in the range of 25 Hz. Obviously, the designer would be well advised to use this figure as a first approximation for stiffness criteria. It is also apparent that very soft support structure as well as structure stiffer than 25 Hz will offer no relief in this environment. This hypothetical curve applies to equipment subjected to the Munson Course alone (aboard an M-35 vehicle). The requirements of other Quality Assurance tests which might be constrained to a given equipment system would substantially alter this picture; the plot serves only to demonstrate the principle.

The natural frequency of beam-like structure may be estimated from the static deflection of the structure under a one-g loading, with the following approximate formula:

$$\text{Natural Frequency} = \frac{\pi}{\sqrt{\text{static deflection}}} \quad (\text{approximately}).$$

The lower plot at right presents a graphic solution for this approximation. If the static deflection is known, an estimate of fundamental frequency may be derived. This concept is presented in greater detail in Volume II, Section 3, and the chapter on "Natural Frequency" in Volume III.

EXCITATION ENVELOPE FOR M-35
VEHICLE OVER MUNSON COURSE
(ASSUMED Q = 10)

Optimum Design Range

G_{rms}

Natural Frequency (Hz) ⟶

APPROXIMATE NATURAL FREQUENCY
BASED UPON STATIC DEFLECTION

Static
Deflection
(Inches)

Natural Frequency (Hz) ⟶
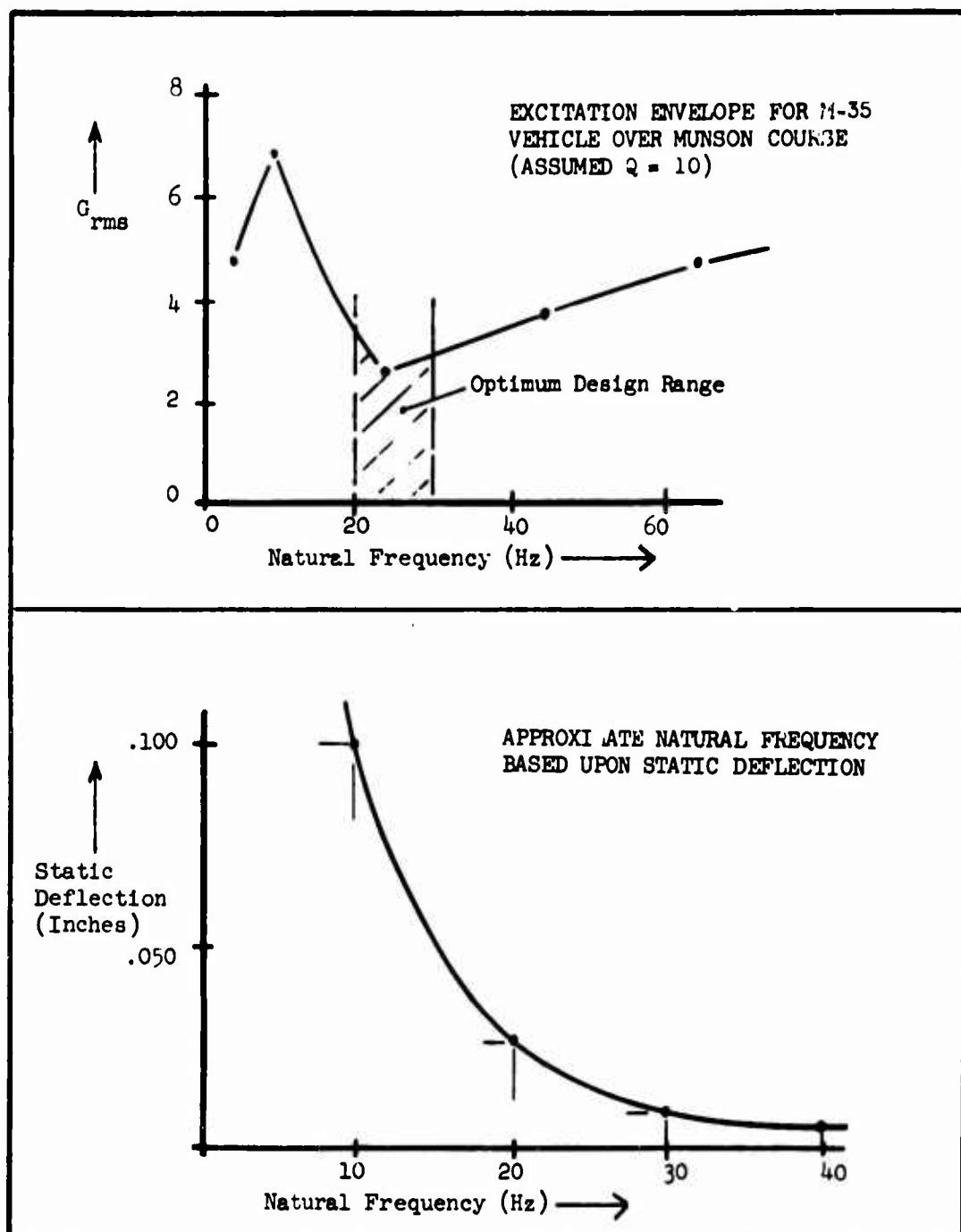
NATURAL FREQUENCY: This structural parameter has a great influence on
the response of an equipment system to the imposed environment.

THE APPLICATION OF DAMPING TO A STRUCTURAL SYSTEM

( )

The degree of damping present in a resonant system determines the extent of the resonant rise, as well as the decay rate of the response curve.

The preceding topics were concerned primarily with those factors that affect the natural frequency parameter of the equipment support structure and its components. A design prerogative of equal importance is the degree of damping that the designer may wish to provide at joints and interfaces and in isolation devices.

The effect of damping in an equipment complex is manifest in the "sharpness" of the response curve, illustrated at right. A lightly damped system exhibits a high response at resonance; the response then falls off rapidly with an increase in frequency input, until almost all the energy is absorbed at 5-10 times the resonant frequency. The sharpness of the peak and the response falloff rate are directly proportional to the amount of damping present in the system.

A highly damped system, alternatively, exhibits a flat response curve. The response buildup at resonance is slight, but the falloff above resonance is also less rapid. The important point here is the design choice of high vs low damping in the structural complex or isolators. The response must be evaluated through the entire range of frequencies contained in the imposed environment. The most damaging energies may occur at frequencies other than resonance depending upon the input intensity.

Damping may be manifest in a structural system by one of three basic mechanisms, or their combination. Internal, or hysterisis damping, is a characteristic measured by the material's ability to store and release energy without loss; viscous damping which is a fluid dynamics phenomena; and Coulomb-friction damping. Air damping is a fourth possibility, which may be classified as viscous since the phenomenon generally follows the laws of fluid mechanics. Internal damping is largely a characteristic of the material used; viscous damping is velocity dependent, reflecting the relative motion between the oscillating body and the resisting body; friction damping is proportional to the force being exerted upon the moving interface and the coefficient of friction between the materials.

The chapter on "Dynamic Attenuation" in Volume III presents a spectrum of attenuation devices arranged in order of the degree of damping present in the systems. The designer may choose an isolating device with varying amounts of damping as well as varying natural frequencies.

Structural damping may often be introduced after the design is fairly well established, without seriously compromising the intended function. Potting, the use of conformal coatings, panel deadening materials, and compliant spacers are examples of this application. Care must be taken to evaluate the impact from other environments on the damping material as well as the component. In general, damping materials are most efficient in shear applications, since they tend to act as a spring in tension-compression situations.
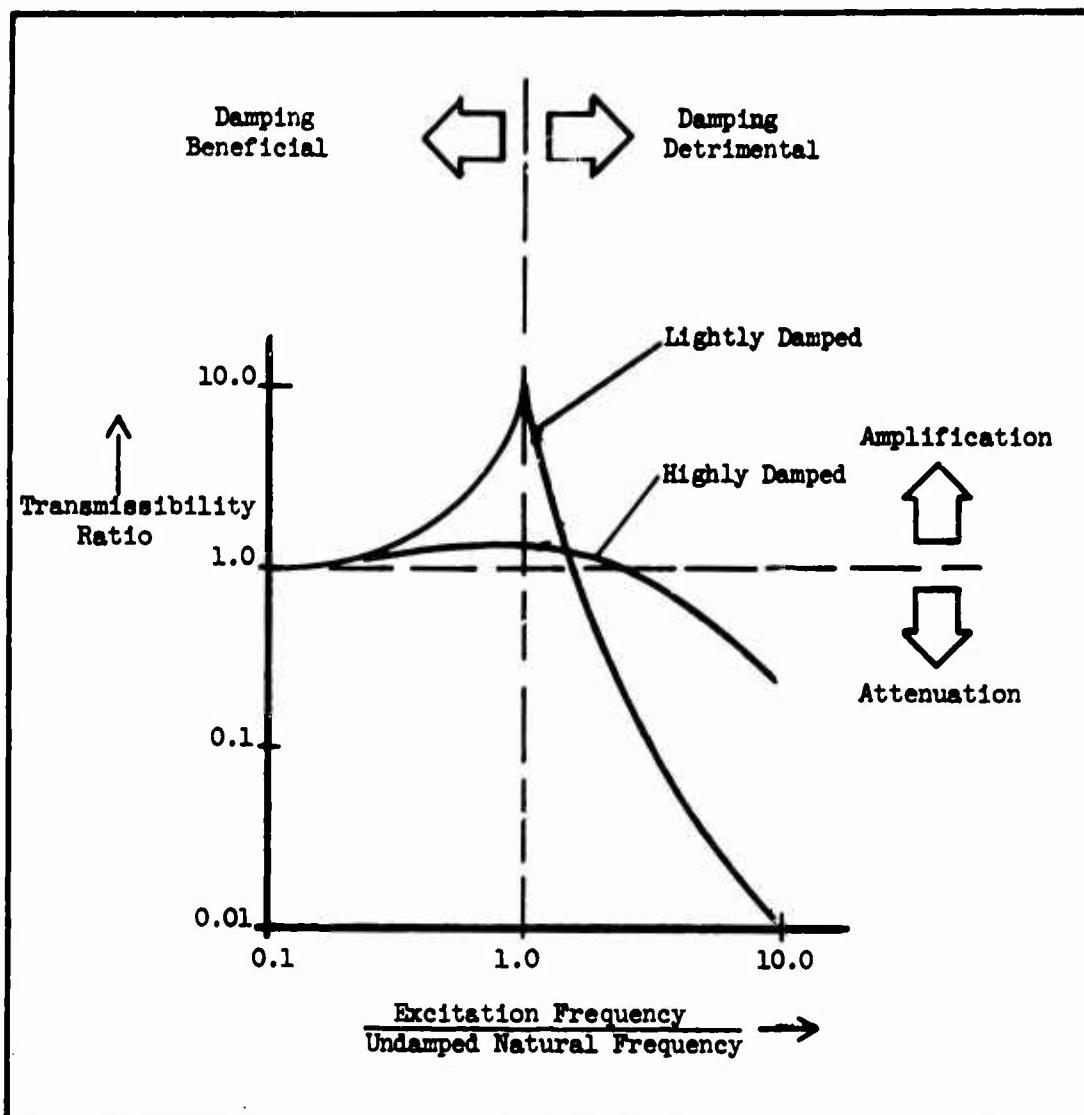
RESPONSE OF A SIMPLE SYSTEM: The effect of damping on a resonating system may reduce the response at resonance, but also increase the response beyond $\sqrt{2} \times f_n$.

MOUNTING FRAGILE COMPONENTS IN EQUIPMENT PACKAGES

Early determination of the fragile elements in an equipment system is an important design prerogative. Some general rules are outlined to implement this task.

The ideal equipment design is one in which all the components are more rugged than the imposed environment. This is, the fragility of all elements exceeds the input excitations. In this ideal situation, the optimum design would consist of a relatively stiff support structure (to minimize dynamic deflections), where each successively smaller element is assigned successively higher natural frequencies.

In the practical situation, there are usually components that will not survive the dynamic environment. A good general rule for both shock and vibration is to hard mount the basic structure, design the support structure for maximum stiffness (high natural frequency), and isolate those elements that will not survive the raw environment. The isolation or mounting frequencies of the fragile components should be about half that of the primary structure. If compromise is necessary in the application of this rule, then follow the concept that component resonant frequency should never coincide with that of the support structure.

The previous generality is most effective in countering the vibration environment, but poses a different problem for shock situations. Bort (8) reports an increasing shock effect in structure exhibiting increased mounting frequency. Thus the lower natural frequency bases would appear to be best for shock applications. This approach, however, causes increased activity from the secondary components, and more possibility of collision damage or overexcursion from the shock. This is particularly true of the ballistic shock, constrained to class 4 and 5 equipments, which is a high acceleration-short pulse duration experience.

The conflict in design philosophy here is apparent. The rule is; consider all of the dynamic influences in preliminary design, evaluate both shock and vibration effects, and get the system into pre-qualification testing as soon as possible.

In the case where most of the equipment elements are under strength for the imposed environment, then isolation of the entire equipment system is the only alternative. In this instance, a stiff support structure is indicated, particularily in the vicinity of the isolators. The components should be stiffly mounted to the basic structure with increasing natural frequency, as the element size becomes smaller. The selection of the isolator characteristics is the subject of the chapter on "Dynamic Attenuation" in Volume III. The rules for optimizing the frequency parameter outlined in a previous topic are applicable.
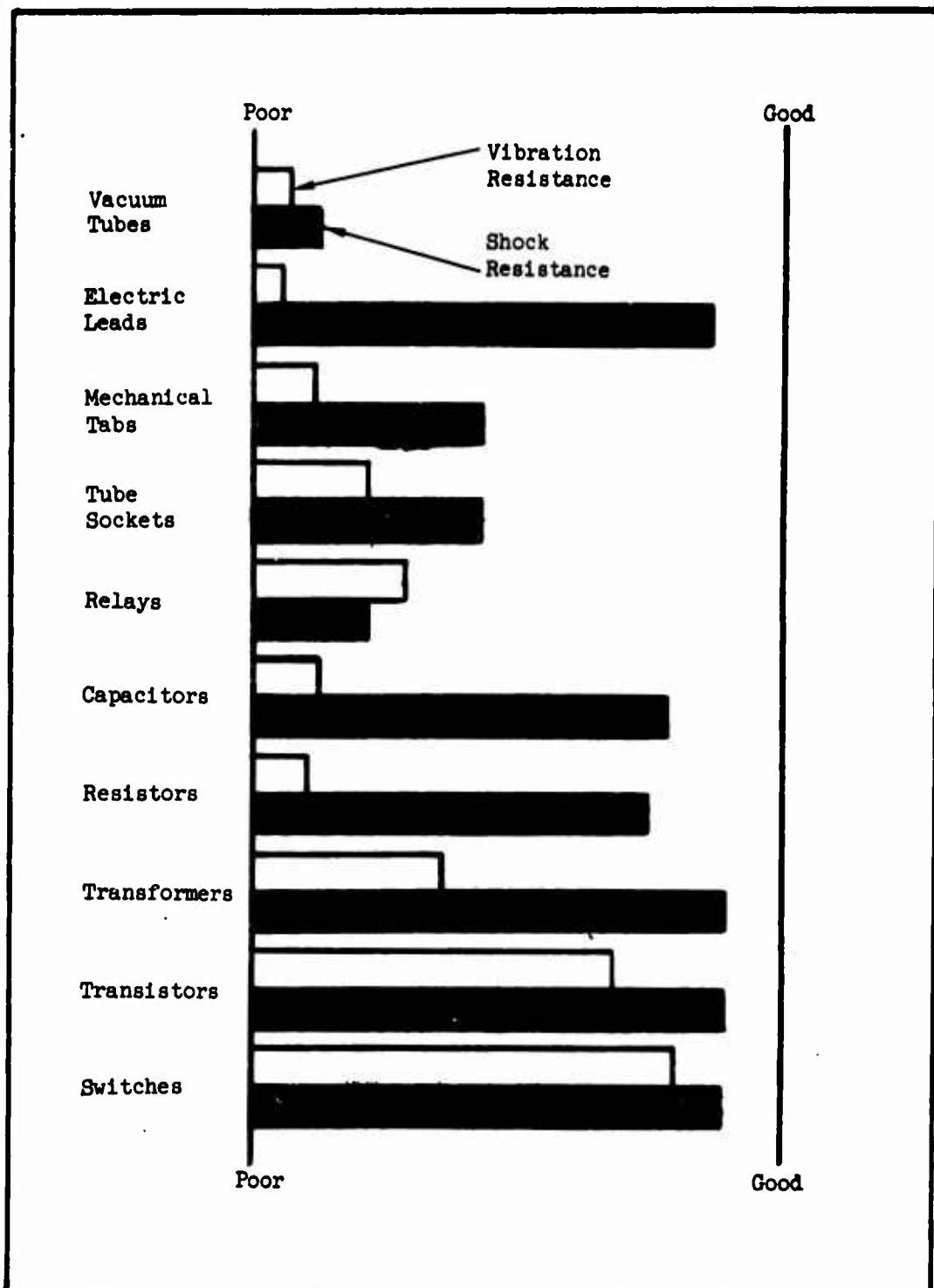
COMPONENT STRENGTH: Determination of the most fragile elements in an equipment system is an important design task. Vibration resistance and shock resistance vary widely for some common electronic components.

A DESIGN OUTLINE FOR IMPROVED STRUCTURAL DYNAMIC INTEGRITY

Some ground rules are offered as a systematic approach to the problem of dynamic integrity in an equipment system. At best, the accepted design procedures are complex, and at times contradictory,

A review of the published philosophies on the design of support structure for shock and vibration environments reveals a variety of design approaches. As stated previously, there appears to be no set solution to the problem. Each situation must be reviewed in the light of the particular environmental stresses and the fragility of the individual items to be housed within the package.

Stiffness and high damping may be the best approach to some dynamic situations, while softness and nominal damping may be more effective for the majority of ground environments. The decision to hard mount or isolate must reflect the individual needs of the fragile components.

If the foregoing paragraphs appear complex and contradictory, then the designer is well into the problem. The design of structure that consistently resists a varying shock and vibration environment is contradictory; the analysis of multi-degree-of-freedom models of the equipment system is indeed complex. The only method that is universally successful is a design based on functional needs which is tested into structural adequacy. This approach is costly and time-consuming.

There are however, some basis steps that the designer may follow which will help him approach the problem in a systematic manner, and assist him in avoiding some of the pitfalls along the way. In the final analysis, any approach is a comparative compromise; a comparison of what the equipment elements will survive versus what the imposed environment has in store, and a compromise on the degree of safety and cost in mass and money that the designer is willing to pay to achieve this survival.

The best background is experience, but experience may not always be extrapolated to new dynamic situations. Nevertheless, the following ground rules are offered as an approach that the designer may follow to improve structural integrity in an equipment system:

1. Structural integrity is a "systems" problem. The Army Quality Assurance provisions are written to qualify complete systems rather than individual elements. The equipment is constrained to function in the field as a unit, rather than a collection of components. The designer must consider all the affecting factors and other environmental stresses. The best functional system in the world is of no value in the field if it cannot be transported there without failure.

2. A thorough understanding of the test and service environment is essential to successful design. Assess the complete loading criteria, and make a composite plot of all the imposed excitations.

3. In general, the environmental intensity for each equipment class increases with increasing natural frequency. Soft structure would then better resist damage, but would also require more space for excursion.

4. Make a one-degree-of-freedom approximation of the equipment structure, and select preliminary structural frequency ranges and damping parameters that best counter the environment.

5. Identify the fragile elements that will not survive the raw shock and vibration excitations. Use dynamic attenuation techniques to protect those marginal elements. Provide ample stiffness at the interface of the element to be isolated.

6. Design for capture rather than support. Dynamic loads are usually omnidirectional. Keep the C.G. of the element within the support pattern to reduce the effect of secondary rocking modes of resonance.

7. Always analyze for both shock and vibration inputs. Any change in the structural parameters required for one excitation will undoubtedly affect the response characteristics caused by the other.

8. After the preliminary loads criteria are established, an expert dynamacist may be helpful in organizing a math model of the structural system to analyze the transfer characteristics of the system. An early estimate of the adequacy of the equipment elements will result.

9. A structural model of the system with masses and stiffnesses represented may also prove effective. The response characteristics at critical locations under the impetus of shock and vibration excitations may be measured in the lab.

10. As the equipment system takes shape, early measurements of energy transfer characteristics will be helpful. These measurements can be taken non-destructively from the actual equipment systems using mechanical impedance techniques.

VOLUME I

METHODOLOGY AND DESIGN PHILOSOPHY

SECTION 4 - USING THE DESIGN GUIDE

- Information Content of the Design Guide

- How the Design Guide Supports an Equipment Development Program

- Information Flow of the Analytical Procedure, Volume III

- Organization of Volume III, Related Technologies

- Application and Limitations of the Analytical Procedure

- Value Decisions Based Upon a Calculated Margin of Safety

- Analytical Procedures for the Evaluation of Structural Capability

- Supporting the Validation Effort of an Equipment System Development Program

- Using the Hardware-Oriented Chapters of Volume III

HOW THE DESIGN GUIDE SUPPORTS AN EQUIPMENT DEVELOPMENT PROGRAM

Volume II of the Design Guide will provide the responsible engineer with an idealized analytical procedure to calculate a margin of safety. Volume III presents a compendium of design oriented disciplines and procedures, aimed at the packaging function.

An equipment development program leading to an Army electronic equipment system will generally encompass five steps in the evolution of a qualified equipment system; issuance of an equipment procurement specification, generation of a system concept, a system design program, prototype fabrication, and a qualification test program. Complete validation of the system will then follow field trials under operational conditions.

Volume II of the Design Guide, "Analytical Procedures", will provide a methodology for establishing first-cut dynamic characteristics of the system after equipment class requirements are defined by the equipment procurement specification. The class parameter will outline the input load spectrum resulting from Quality Assurance Provisions. The system concept effort will in turn, provide preliminary structural characteristics to the analysis chain, after functional parameters and gross system characteristics are defined from the procurement specification. Thus, the analysis procedure outlined in Volume II will operate on the input load criteria and first-cut structural definition to establish preliminary structural boundaries for the formal system design program. As the system is more clearly defined, an analytical iteration will take place to firm the structural parameters. After the first system prototype is fabricated, the actual energy transfer characteristics can be established by measurement of mechanical impedance. Following design changes that may be incorporated into the first system, the analysis procedure provides for final safety margin calculations in support of equipment qualification.

Volume III of the Design Guide, "Related Technologies", provides a background of the mechanical disciplines needed to support the equipment development program. Although the chapters of Volume III are all design oriented, the information may be categorized into three main areas; an analytical group which will support the calculations outlined in the analysis procedure; a group of chapters directed towards the equipment design program, and a group of chapters arranged to support the test and validation phase of the program.

Since all of the sections of Volume III are concerned with packaging Army electronic equipments for the shock and vibration environment, there will naturally be some cross-over in the use of the material. The chapter on Stress Concentration for example, will be useful to the packaging engineer during the design, analysis and test phase of the equipment program. The same is also true for Volume II; the analytical procedures outlined will support the entire equipment program, from preliminary design through qualification.
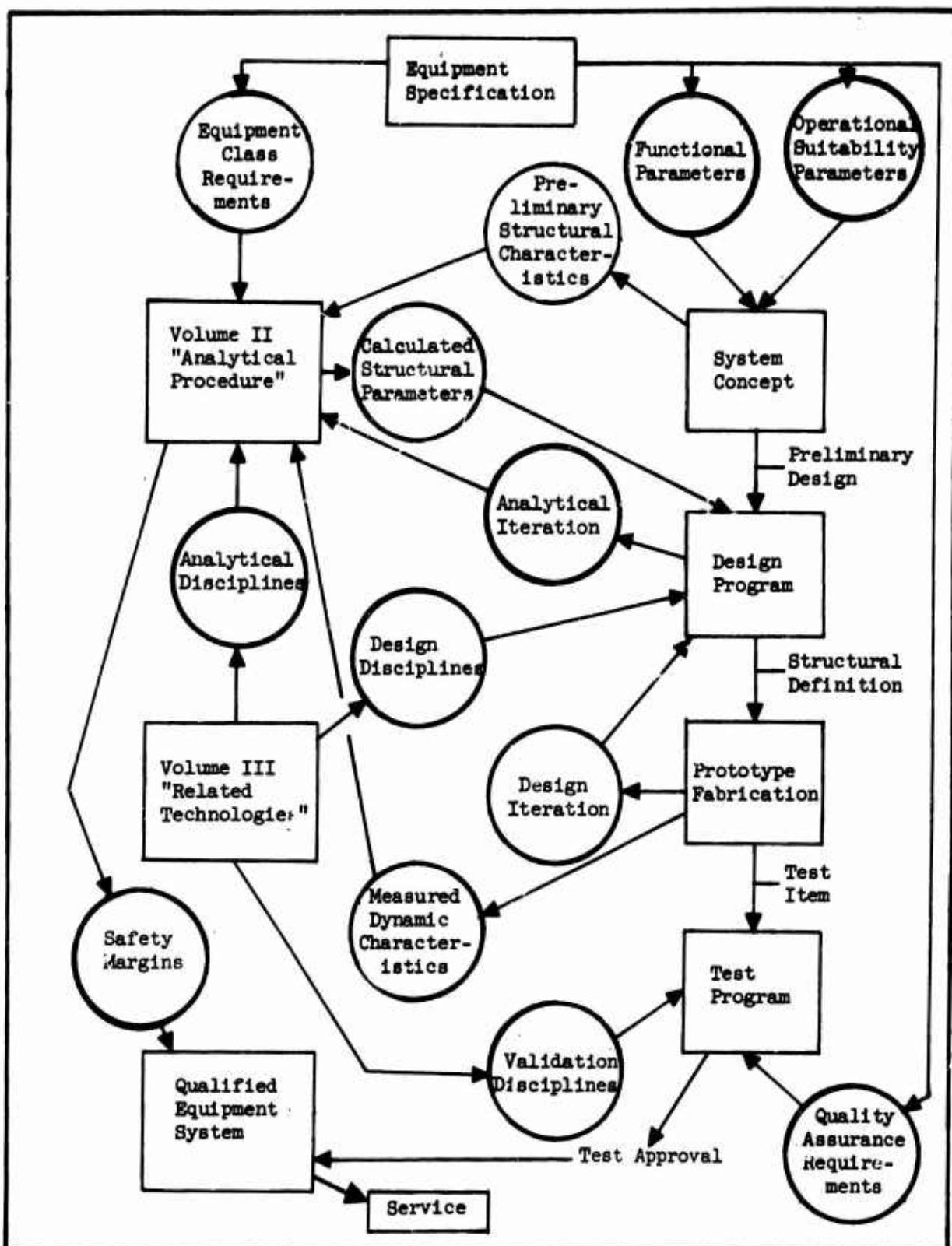
SUPPORTING THE EQUIPMENT DEVELOPMENT PROGRAM: The guide will aid in the development of an equipment system, procurement specification through system qualification.

INFORMATION CONTENT OF THE DESIGN GUIDE

The Shock and Vibration Technical Design Guide is a collection of design-relevant treatises on the design, analysis, and fabrication of electronic equipment systems constrained to survive the Army service environment and to be qualified by a unique set of required dynamic tests.

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┌─────────────┐                    ┌──────────────────────────┐
│   SV/TDG    │                    │ Equipment Level Criteria │        ⬤
│  Volume I   │                    │ System Packaging Concepts │   Equipment
│             │ ──────────────┐    └──────────────────────────┘    System
│"Methodology and│            │              ▲
│ Design Philosophy"│         └──► ┌──────────────────────┐
└─────────────┘                    │"Optimum Mechanical   │
        │                          │ Packaging Handbook"  │
        │                          └──────────────────────┘
        ▼
┌───────────────────────────────┬────────────────────────────────┐
│ ● Roadmap through the Design Guide │ ● Packaging philosophy      │
│ ● Designing for dynamic integrity  │ ● Quality Assurance provisions │
│ ● Equipment class categories       │ ● Optimizing component location │
│ ● Limitations inherent in testing  │ ● Controlling dynamic environment │
└───────────────────────────────┴────────────────────────────────┘
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┌─────────────┐
│   SV/TDG    │                                         ⬤
│  Volume II  │                                     Equipment
│             │                                      System
│ "Analytical │ ──────────────────────┐
│  Procedures"│                       │
└─────────────┘                       │
        │                             │
        ▼                             │
┌─────────────────────────────────────────────────────┐
│ ● Shock and vibration input spectrum by equipment class │
│                                                         │
│ ● One D-O-F Idealization                                │
│                                                         │
│ ● Structural Response                                   │
│                                                         │
│         Random                                          │
│         Sinusoidal                                      │
│         Shock                                           │
│                                                         │
│ ● Strength Calculation                                  │
│                                                         │
│ ● Design Evaluation - Margin of Safety                  │
│                                                         │
│ ● Illustrative Examples                                 │
│                                                         │
│         New Equipment Concepts                          │
│         Existing Equipments                             │
└─────────────────────────────────────────────────────┘
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INFORMATION FLOW OF THE ANALYTICAL PROCEDURES, VOLUME III

The analytical procedure is essentially a response calculation which designs a force related number by factoring the imposed input with an energy transfer function.

The analytical procedure offered in this design guide is presented on the premise that a value decision relating the dynamic integrity of a structural element may best be expressed as a number, or safety margin. The margin of safety is calculated by comparing the anticipated dynamic response of the element with its measured or calculated fragility. The safety margin is thus a numerical comparison of dynamic strength with dynamic load.

The essence of the response calculation is the definition of a force related number (for easier comparison with fragility) by factoring the input disturbance with an energy transfer function; that is, Response = Input x Transfer Function. The input excitations, as previously indicated, are a matrix of static equivalent accelerations resulting from the required Quality Assurance tests. These inputs are arranged as to category, (steady-state, random, and shock loads) and are designated by the equipment specification. The transfer characteristics are largely dependent upon structural parameters (such as natural frequency and damping) which are measured, calculated, or estimated from the equipment system. It follows that the better the estimate, the more confidence may be expected from the response calculation.

A limitation in the accuracy of the response is inherent from factors in the equation. Input excitations are defined for the total system only, since the tests are required for entire equipment group. Inputs are measured or defined at the interface of the test machine with the equipment base, providing system rather than component loading information. Furthermore, some impedance feedback of specimen to machine is likely in some of the Quality Assurance tests, making complete load definition difficult.

The transfer characteristics are also subject to some speculation, particularly in the preliminary design effort where very little phase information exists to describe the dynamic relationships between individual spectural elements. As the design progresses to a well simulated model, better information may be calculated and measured.

A general model of the analytical procedure is illustrated in the accompanying figure. The first line of design information is derived from the equipment specification. Contractural requirements of equipment class and resulting Quality Assurance tests lead to a spectrum of input loads. The specification also provides the first-cut system concept parameters, leading to preliminary estimates of energy transfer character'stics. The calculated structural response in turn provides one element of the comparative analysis.

The resistive capability of the equipment element is then factored into the comparison by measuring, calculating, or estimating the equipment fragility. The resulting comparison leads to a safety margin which may be accepted, rejected, or refined as the case dictates. If the alternatives are refine or reject, then the entire analysis loop must be

iterated. Changes in the structural characteristics to improve fragility for example, will usually also affect the elements resonant parameters, transfer characteristics, and ultimately response; a new safety margin calculation is then needed.
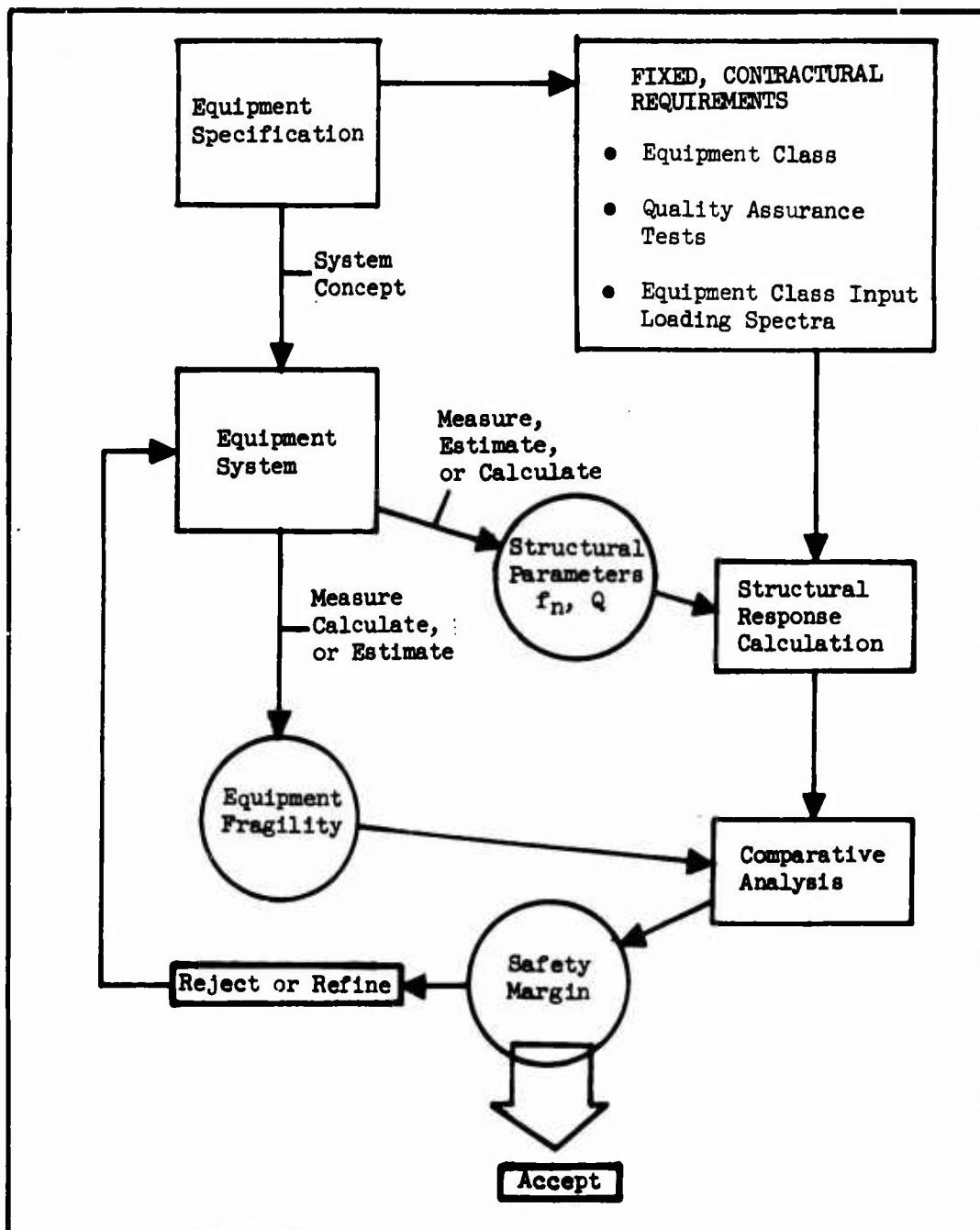
ANALYTICAL PROCEDURE: An iterative process is offered to evaluate the response of an equipment structure to the imposed dynamic environment.

ORGANIZATION OF VOLUME III, "RELATED TECHNOLOGIES"

The design-relevant information presented in Volume III is organized into three functional categories; the mechanical engineering disciplines of analysis, validation, and hardware application.

Volume III of the Design Guide, "Related Technologies", is a compendium of design aids, technical details, and mechanical disciplines that are useful in structuring the electronic equipment package. The information is presented in straight mechanical engineering terms, and is generally oriented for the structural design engineer. There are sections of the Volume that will be useful to other technical disciplines related to the design of structure for the dynamic environments; the Project Engineer, loads and stress analysts, test engineer, as well as the equipment packaging designer will find information of use in performing his particular speciality.

The elements of Volume III are categorized into three areas of roughly similar technical content; analysis, validation, and hardware. There will of course be much cross-over of information between the categories. For example, the analyst and the designer will both be vitally interested in stress concentration, while the stress analyst and test engineer will be concerned with instrumentation problems. The program manager or project engineer will probably be interested in all of the fields, since a general knowledge of all the technical details is vital to his decision making processes.

The analytical section of Volume III will include chapters on basic mechanics, natural frequency, fatigue, stress concentration, and mechanical impedance. The second category, hardware, is typically concerned with the nuts and bolts aspect of the packaging problem, and includes chapters on dynamic attenuation, materials and processes, and mounting and installation techniques. The group of chapters concerned with validation are test oriented, and include dynamic simulation, instrumentation, and fragility.

The intent of each of the chapters in Volume III is to acquaint the reader with the language of the particular field, and summarize the knowledge of the techniques in abstract form. A bibliography is included in each chapter which not only supports the thesis of the chapter, but also will offer an annotated reference list on the subject for study in greater depth.

Each of the chapters is introduced in context with the equipment packaging problem for the unique Army shock and vibration environments. The appendix of each section includes a glossary and symbology list, and a collection of handbook type data of use to the mechanical engineer.

Some of the technical material offered in the "Related Technologies" volume will appear rather basic and somewhat tutorial to an engineer who is expert in that area. The scope of the material presented in this volume is intended to acquaint the reader with the discipline, to review the material for those who have been away from school for a number of years, and provide a common base of language from which further study can build. Many technical people concerned with packaging have matriculated from the electrical fields, and thus have a great need for information

in the mechanical disciplines. Similarily, structural engineers may often be weak in instrumentation, isolation, testing, or metallurgy. The philosophy of this volume is then, to sum the diverse technical skills needed to accomplish the equipment packaging task.

VOLUME III — RELATED TECHNOLOGIES

Analytical

- Basic Mechanics
- Natural Frequency
- Fatigue
- Stress Concentration
- Mechanical Impedance

Hardware

- Dynamic Attenuation
- Materials and Processes
- Packaging Design Techniques

Validation

- Dynamic Simulation
- Instrumentation
- Fragility

VOLUME III, RELATED TECHNOLOGIES: A compendium of design aids, technical details, and mechanical disciplines that are useful in structuring the electronic equipment package.

APPLICATION AND LIMITATIONS OF THE ANALYTICAL PROCEDURES

The analysis approach is based upon the response characteristics of an idealized single-degree-of-freedom model for which the natural frequency and resonant rise is known or can be satisfactorily estimated.

The intent of the analytical procedure presented in Volume II is to provide the packaging designer with an approach to the numerical evaluation of an equipment structure and its components. The elements of this analytical process are inputs, transfer functions, responses, fragility, and the calculation of a safety margin. Stated in general terms, a procedure is defined to assess the dynamic strength of the structural element and compare this value with the anticipated environment.

The source of numerical loads for system input, the start of the analysis chain, are the excitations resulting from the required Quality Assurance tests. The details of these loading criteria were discussed in depth in a preceding section of this volume. There are important limitations to these input loads which the analyst must recognize; the loads are defined on a system basis for a specimen of indeterminate resonant characteristics. As the actual equipment structure varies from the norm, so do the exact values of the input excitations.

The idealization of the complex system with a single degree-of-freedom model has certain inherent limitations. The approach does however, offer a simplified procedure for defining the first-cut responses of the complex system. The determination of dynamic strength which results from this calculation is a reasonable approximation of the first resonant mode of the equipment system. Disastrous failu s most often occur due to the first resonant peak of the system. Then, as the system structure becomes better defined and the prototype model is fabricated, the actual transfer characteristics may be measured by impedance techniques. This non-destructive measurement will reflect the exact phasing relationships within even the most complex structure.

The analysis approach is based on the response characteristics of the element for which the resonant frequency and resonant rise are known or can be adequately estimated. The input loads are expressed in terms of an equivalent static force or acceleration plotted against natural frequency. Comparison of fragility with calculated response, leads to a value decision based upon the safety margin, a concept discussed in detail in the following thesis.

An important feature of the expression of inputs in force related numbers is the use of frequency-domain plots. The response of the structural element to a given input, is dependent upon natural frequency and damping. Further, all input excitations are then viewed on the same scale, an approach which allows easy determination of the most critical frequency ranges for a given equipment class. This knowledge alone is helpful to the designer in selecting stiffness ranges and isolation frequencies which will be optimum for his equipment.

Elements of the Analytical Procedures	Some Important Limitations
Input Excitations	Uncertain Test Load Inputs System Level Only
Response Characteristics	One D-O-F Approximation Difficult to Estimate Complex System
Fragility Comparison	Little Data Available During Preliminary Design Phase
Safety Margin	Validity Dependent Upon Previous Analytical Elements
Value Decision	Validity Dependent Upon Previous Analytical Elements

THE ANALYTICAL PROCEDURES: A numerical evaluation of the structural adequacy of the equipment package.

VALUE DECISIONS BASED UPON A CALCULATED MARGIN OF SAFETY

The intent of the analytical procedure is to provide the Mechanical Engineer with a method of calculating a safety margin. Follow-on procedures are based on this margin, which may be negative, positive, or borderline.

The elements of the analytical procedures outlines in Volume II are pointed towards the last comparison process in the chain, the value decision on the structural adequacy of the element in its intended dynamic environment based upon a calculated margin of safety. This calculation chain is, in practice, an iterative procedure, and is usually upgraded as better information becomes available to the analyst.

The calculated safety margin will conveniently fall into one of the following groups; acceptable, very negative, very positive, or a borderline (marginal) situation. The very positive or very negative margins are critical and demand immediate attention by the analyst and structural designer. The net effect in the equipment system is an overweight penalty in an excessively positive margin situation (which is intolerable in modern mobility concepts), or a dynamically weak structure in the case of the negative margin, leading ultimately to a service failure. The intermediate case, or borderline margins are often the most demanding since no clear-cut action is indicated and the fixes are usually more subtle and less responsive. Acceptable margin implies sufficient dynamic integrity, and the design is left as is.

Safety margins which are very negative usually occur only in the preliminary design phase, with the exception of a gross oversight. In most cases, direct and usually severe action is indicated, since service failure is almost inevitable. Some of the first corrective design steps include a strengthening or stiffening of major support structure to avoid any damaging resonant situations; or isolation of the most critical fragile components. Some of the more drastic design changes precipitated by very negative margins include: isolation of the entire equipment system; a complete redesign of the major structural elements; restriction of the system operational environment to reduce the anticipated excitation severity; or a downgrade in the equipment class to a less demanding category. All of these alternatives are expensive and constitute a major change in the system structural concept. Obviously, highly negative margins should be avoided.

Very positive margins alternatively, do not usually cause catastrophic failures but more often result in system operational degradation due to overweight. Some of the most effective design changes indicated in this case are: the use of lighter alloys for major structure, such as magnesium and aluminum substituted for steel shapes; reduction of the number of structural interfaces, which has the effect of reducing the number of joints; design of major structure at a constant stress (constant margin) using a consistent material for maximum material efficiency; reduction of the number of isolated components wherever practical, to reduce the weight expenditure for structural interfaces and isolation units; upgrade the operational category or equipment class.

In those marginal cases where the safety factor is minimal, certain design steps may be taken to improve the situation without a major rework. These approaches include: local stiffening of critical structure,

particularily joints; detuning of resonant structure by adding masses
and springs; damping of vibrating panels and elements to reduce resonant
rise; isolation of critically fragile elements; and change in the mode of
support to alter the resonant frequency of the element.

VERY NEGATIVE MARGINS

● Complete Redesign

● Isolation of Entire System

● Strengthen Major Structure

● Isolate Critical Fragile Components

● Downgrade Operational Category

● Restrict System Operation

VERY POSITIVE MARGINS

● Check for Constant Stress/ Stiffness

● Reduce Structural Interfaces

● Hand-Mount Elements Where Practical

● Use Lighter Alloys

● Upgrade Operational Category

BORDERLINE MARGINS

● Local Beef-Up of Major Structure

● Isolate Most Fragile Elements

● Detune with Added Mass

● Support Alteration for Mode Change

● Damp to Reduce Resonant Rise

● Stiffen and Damp Joints

MARGIN OF SAFETY: Alternative courses of action are indicated for the
varying margin situations.

ANALYTICAL PROCEDURES FOR THE EVALUATION OF STRUCTURAL CAPABILITY

An analytically oriented category of chapters is presented in Volume III to assist the evaluation effort. These chapters include treatises on Basic Mechanics, Natural Frequency, Fatigue, Stress Concentration, and Mechanical Impedance.

The analytically oriented group of chapters within Volume III are treatises on Basic Mechanics, Natural Frequency, Fatigue, Stress Concentration, and Mechanical Impedance. Although the material presented in this group is primarily directed towards the design and evaluation of new equipment concepts, the material will also be useful during the validation of equipments and the analysis of failed elements for fault isolation and correction.

The chapter on mechanics is intended to be a refresher on the mechanical disciplines of statics, dynamics, and mechanics of materials. It is assumed that the reader has had some course work in these areas and is basically familiar with the material. The chapter will then serve to peak-up his knowledge, review the approaches and terminology, and generally provide the procedures for evaluating a complex structure. The material covered includes the concepts and methods used in force analysis of static structures, the determination of stresses in members, and the dynamics of vibrating systems.

The mechanics chapter will support the analytical procedures outlined in Volume II, and will find utility with all the technical people interested in effective packaging; the Project Engineer, structural designer and analyst. The material is heavily stress oriented and leads into the evaluation of stress effects, and hence safety margin. Since the analysis chain is an iterative process (as the dynamic parameters pass from the conceptual to the hardware phases) the disciplines outlined in the mechanics chapter will be applicable throughout the evolution of the equipment system.

Moving one step further into the detail of a structural analysis aimed at dynamic integrity, the analyst and designer will be concerned with the parameter of natural frequency. The chapter on natural frequency will establish the importance and limitations of the resonant parameter, and discuss in detail the application of fundamental frequency to balanced structural design. In addition, this chapter will provide ground rules for the measurement, estimation, and calculation of natural frequency. The intent of this chapter is to outline, for the designer, the major factors affecting fundamental frequency, and the usual methods for shaping the parameter into more efficient structure. Since natural frequency does reflect the important structural elements (such as support mode, deflection characteristics, and material elastic modulus), the designer must understand their impact and use these elements to his advantage.

The phenomenon of fatigue is of vital importance to structure which will be subjected to dynamic environments. Excessive vibratory stimulus usually results in a fatigue failure of the material; in some cases where repeated shock loadings excite a structure at resonance, a fatigue failure may also be induced. The chapter on fatigue addresses the basic design choices available to the equipment package engineer; improve element strength or reduce the dynamic stress effect of the structure exposed to fatigue conditions. This section discusses the nature of fatigue, the major factors affecting fatigue strength, and the accepted analysis

procedures for structural fatigue evaluation. The fatigue chapter also outlines the methods for the application of fatigue technology to equipment packaging problems, and the design techniques for the improvement of fatigue resistance in support structure.

Most failures of engineering materials emanate from an incipient flaw or stress discontinuity within the structure or material. These stress magnification effects are known as stress raisers and are evaluated by stress concentration factors. The chapter of Volume III devoted to stress concentration is intended to establish the importance of the stress raiser effect to the design of equipment packages, and generally outline the influence of stress concentration on dynamic structural integrity. Data is presented which evaluates the concentration effect in terms of the theoretical factor, the fatigue stress concentration factor, and material notch sensitivity. Stress amplification in structural members is illustrated by strength calculations for elements subjected to static, impact, and repeated loads. Some rules and suggestions are outlined to reduce the stress amplification effect in equipment structure.

Mechanical impedance techniques represent a new approach to the evaluation of structural response in complex systems. Impedance analysis provides a methodology for the numerical evaluation of the coupling effects and energy transfer characteristics of dynamic excitations in multi degree-of-freedom systems. The chapter on mechanical impedance introduces the reader to the basic concepts, language, and limitations of the approach and outlines the electrical analogy of impedance. The impedance characteristics of mechanical systems are discussed and the concept of four pole system analysis techniques is outlined to assist the analyst in using this tool. The experimental possibilities of mechanical impedance evaluation of complex systems is offered as an approach to energy transfer analysis, one factor in the analysis procedure presented in Volume II. The application of the technique to practical hardware is offered for cases involving shock attenuation, vibration isolation, and noise control.

VOLUME III — DESIGN ANALYSIS CHAPTERS

- Basic Mechanics

- Natural Frequency

- Fatigue

- Stress Concentration

- Mechanical Impedance

VOLUME III: The initial set of chapters in this volume of the Design Guide, will review the analytical disciplines important to the evaluation of equipment structure.

SUPPORTING THE VALIDATION EFFORT OF AN EQUIPMENT SYSTEM DEVELOPME... PROGRAM

A category of test oriented chapters are presented to assist the design engineer through the validation phase of his program. These chapters include treatises on Dynamic Simulation, Instrumentation, and Fragility.

The success of an equipment development program is generally measured in terms of the operational suitability of the system. One of these operational suitability parameters is the structural adequacy of the system during service environmental influences. The Quality Assurance provisions are directed towards the demonstration of this structural adequacy under loading duress, particularily shock and vibration. Thus, the shock and vibration tests are conceived to demonstrate the dynamic structural integrity of the equipment system during accelerated and simulated service conditions. To assist the designer-analyst, and responsible equipment program manager through this phase of the equipment development program, a group of chapters is presented in Volume III dealing with the validation of the equipment system. These chapters include treatises on Dynamic Simulation, Instrumentation, and Fragility.

The Dynamic Simulation chapter describing the actual testing aspect of the validation effort, is divided into two categories of shock and vibration testing: developmental testing, which is largely exploratory and is designed to establish the structural parameters of the system to assist the analysis procedures, as well as establish a confidence that the system will in fact pass the required tests; and the Quality Assurance tests, which are a contracturally required group of tests, reflecting the needs of a given equipment class. The intent of this chapter on dynamic testing is to present some of the qualitative aspects of the range of tests unique to Army electronic equipment validation. Details are given on the physical description of the individual tests, such as information of the test machines, fixturing needs, test parameters, failure or acceptance criteria, and some of the important functional and physical limitations of the test. The primary concern of the designer and analyst is the dynamic excitations resulting from the test spectrum. Accelerations peculiar to the individual tests are plotted in the frequency domain to illustrate the damage potential inherent in the experience. The summation of these inputs, by equipment class, is the basis for the design criteria presented in Volume II, "Analytical Procedures." Some design-relevant observations are presented for each of the individual tests, including information on the pulse shape, load iteration, and random aspects of the dynamic stimuli.

A frequently misused and often misunderstood aspect of the testing disciplines is the science of instrumentation. Instrumentation, as it is presented in Volume III, deals with both data acquisition and data reduction. In the testing business, a great deal of data is usually accumulated for a range of tests. The subsequent value of this information to the equipment designer is more often than not lost due to a variety of influences, most of which could be avoided by proper pre-test planning.

Some of the important decisions that must be made by the responsible engineer prior to the test include: an overview of the end-use of the information, including the desired format of the data; the type and range of the transducer to be employed; the location of the transducer on the

test specimen (this point alone accounts for a good deal of lost data);
and a good idea of the parameters of the peripheral gear to be used with
the transducers, such as the degree of signal conditioning and filtering
that will be employed. The central theme of the Instrumentation chapter
is aimed at providing answers to these and other questions concerned with
dynamic data provisions, in a language understandable to the mechanical
engineer. A good portion of this chapter deals with the manual and auto-
matic data reduction techniques currently available to the structural
engineer, and how he may more efficiently use the facilities for improved
structural integrity.

The Fragility chapter is directed towards the problem of equipment fail-
ure, and how the failure surface may be expressed in terms useful to the
mathematical process. The resistive capability of an equipment element
to shock and vibration excitations is one half of the analytical process
leading to a safety margin. This chapter addresses the problem of defining
this surface in engineering terms.

The failure modes of various equipment levels is discussed, along with the
fracture characteristics of common engineering materials. Equipment fail-
ures are classified into two basic groups; failure by the first influence
of load, or failure due to the repeated application aspect of load, such
as vibration. Failure may be a fracture or fatigue of the base material,
an elastic instability, an over-excursion or bottoming, or a functional
failure or degradation following the dynamic stimulus. The details and
differences inherent in these failure modes are discussed in this chapter,
and the design inferences and importances outlined.

VOLUME III – CHAPTERS ON THE VALIDATION OF EQUIPMENT SYSTEMS

- Dynamic Simulation

- Instrumentation

- Fragility

VOLUME III: A group of chapters are presented to assist the designer and
Project Engineer through the validation phase of an equipment development
program.

USING THE HARDWARE-ORIENTED CHAPTERS OF VOLUME III

Dissertations on Dynamic Attenuation, Materials and Processes, and Packaging Design techniques are directed towards the packaging engineer.

The last group of chapters in volume III deal basically with hardware disciplines and are thus directed towards the packaging engineer and the problems of support structure for the equipment package. The group contains summary treatises on Dynamic Attenuation, Materials and Processes, and Mounting and Installation Techniques.

The Dynamic Attenuation chapter deals with the array of devices available to the designer for the control of the dynamic environment within the equipment itself. The concept of attenuation is developed as a means of categorizing the common energy absorption techniques; isolation, absorption, and damping. A spectrum of these devices is cataloged on the basis of response of the attenuator, an independent variable which relates the resonant rise of the device. The selection procedure offered in this chapter matches the anticipated environment with the component fragility, to arrive at an allowable transmissibility curve. This plot then displays the frequency ranges that are critical, and provides the basis for the selection of a suitable attenuating device. The response spectrum is supported by a discussion of the mechanical characteristics of the materials employed in the available attenuation devices. A section on the details of current attenuation techniques is featured in this chapter. The approaches and hardware recommended in this section are based upon an arbitrary equipment "level" classification, developed in the Optimum Mechanical Packaging Handbook.[6] The equipment levels used are component, sub-chassis, chassis, and console, and include discussions on techniques of isolation and absorption as attenuation procedures.

The Materials and Processes chapter provides an overview for the mechanical designer on the optimizing of packaging materials, their heat treat processing, and the array of manufacturing processes that may be applied during fabrication of the equipment system. The concept of balanced design is introduced; the numerical comparison of the stress induced in the equipment from the dynamic environment with the resistive capability of the fabricated structure, or dynamic structural integrity. The materials portion of this chapter deals with the important physical properties of engineering materials, how these properties are determined experimentally, and the characteristics of the metallurgical mechanisms influencing strength, such as wear, surface effects, and alloying elements. A summary section on materials presents some guidelines for the selection of an optimum material for a given structural application. The alloys cataloged include the ferrous alloys, aluminums, magnesium, copper and nickel alloys, beryllium, titanium, and some common plastics. The manufacturing processes are categorized into three groups; joining procedures, forming procedures, and material processing procedures. All of the important fabrication processes may be lumped into one or more of these categories. The emphasis of this chapter is directed toward the physical aspect of materials and manufacturing procedures as they relate to dynamic structural integrity. The scope of the information and data covered summarizes the physical material characteristics for the designer to promote more effective equipment packaging. Some metallurgical and

fabricational problems will necessarily fall outside of these boundaries, but will be recognizable to the non-expert so that help may be sought.

The chapter on packaging design techniques will probably by the most often used of Volume III by the mechanical packaging engineer. The subject matter covers the nut and bolt aspect of effective equipment packaging. This chapter will supplement the preceding section of Volume I on packaging methodology, since the design approaches and philosophies developed there will be implemented in the mounting and installation chapter. Some of the details presented include design tips for the support of fragile components, the application of isolation and absorption devices, the design of interfaces and attachments, designing for proper accessibility, the impact of reliability and other operational suitability requirements, and some design constraints associated with the application of engineering materials and manufacturing processes.

<u>VOLUME III — HARDWARE ORIENTED CHAPTERS</u>

- Dynamic Attenuation

- Materials and Processes

- Packaging Design Techniques

VOLUME III: This volume also presents a group of chapters concerned with the hardware aspect of equipment packaging.

VOLUME I

METHODOLOGY AND DESIGN PHILOSOPHY

SECTION 5 - APPENDIX

- Bibliography

- Typical Wheeled Vehicles

- Typical Track-Laying Vehicles

- Examples of Class I Through VI Equipment

- Mounting and Transport Illustrations

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TYPICAL WHEELED VEHICLES
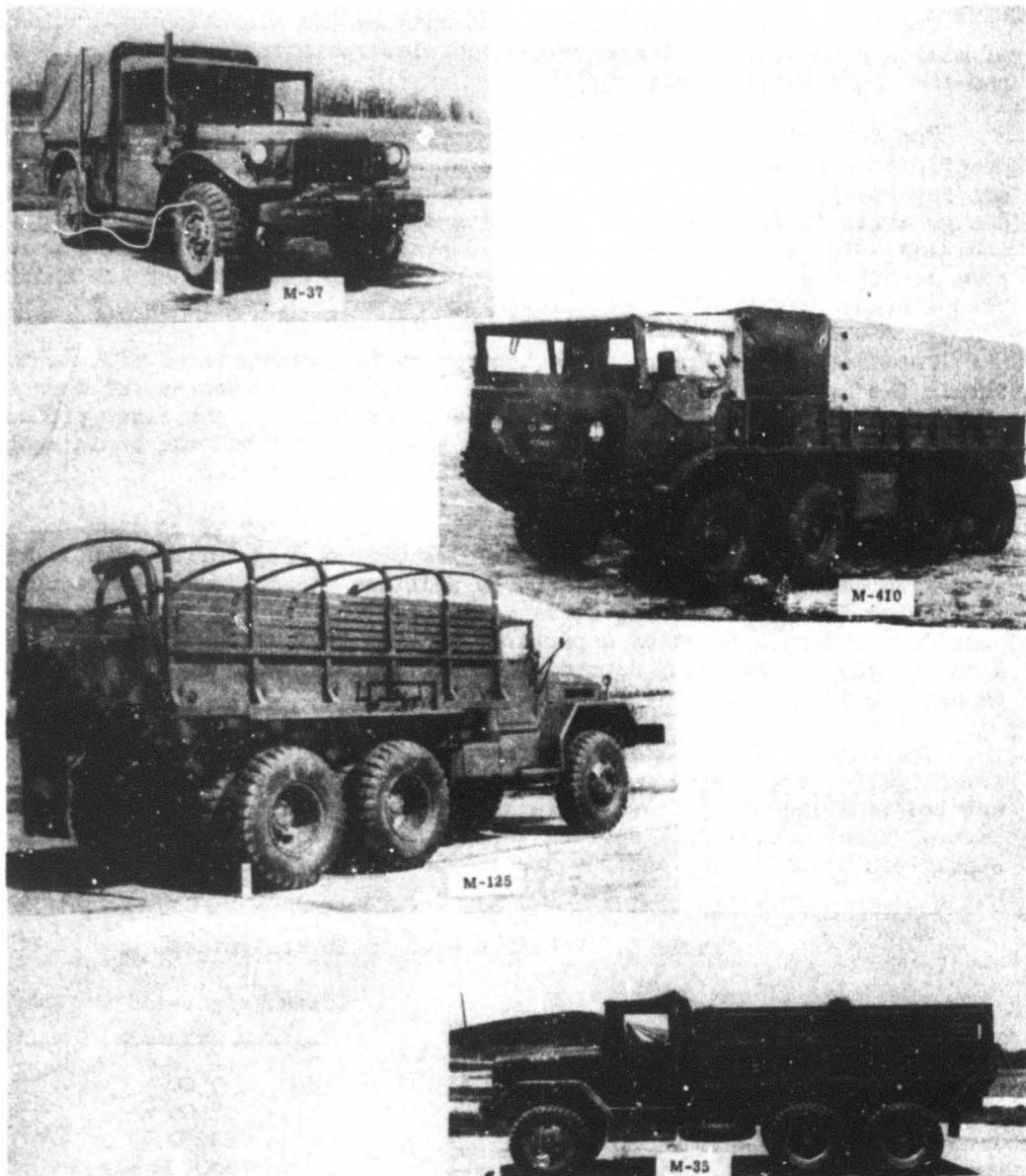
Vehicles	Gross Weights (lb)
Truck, cargo, 3/4 ton, 4x4, M37	7,500
Truck, cargo, 2-1/2 ton, 6x6, M35	17,500
Truck, cargo, 2-1/2 ton, 8x8, M410	13,750
Truck, cargo, 10-ton, 6x6, M125	52,000

TYPICAL TRACK-LAYING VEHICLES

M-116

M-56

M-113

M-41

M-60
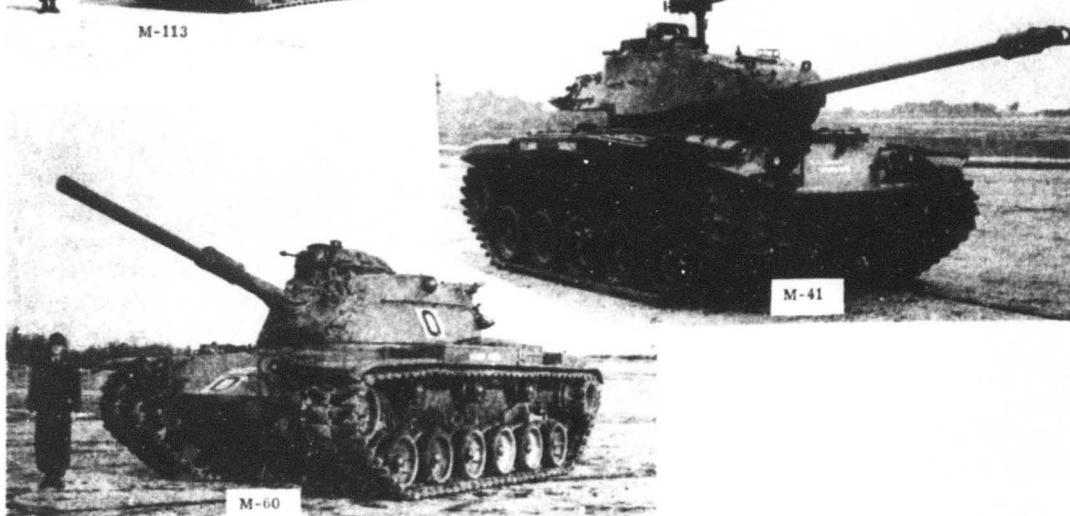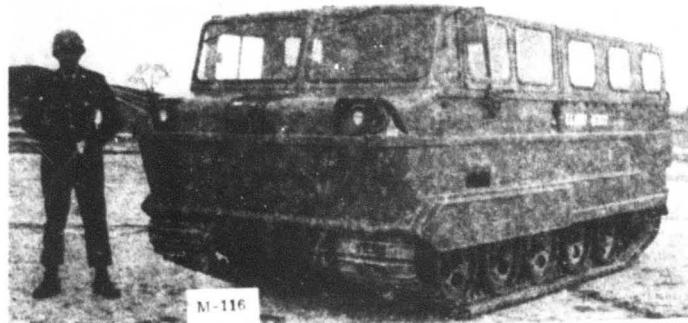
Vehicle	Gross Weights (lb)
Carrier, personnel, T116	10,500
Carriage, gun motor, M56	15,000
Carrier, armored, personnel, M113	23,000
Tank, 90-mm gun, M41	52,000
Tank, main battle, 105-mm gun, M60	103,000

EXAMPLES OF CLASS I THROUGH VI EQUIPMENT

CLASS I EQUIPMENT: Transported as loose cargo or manpacked.
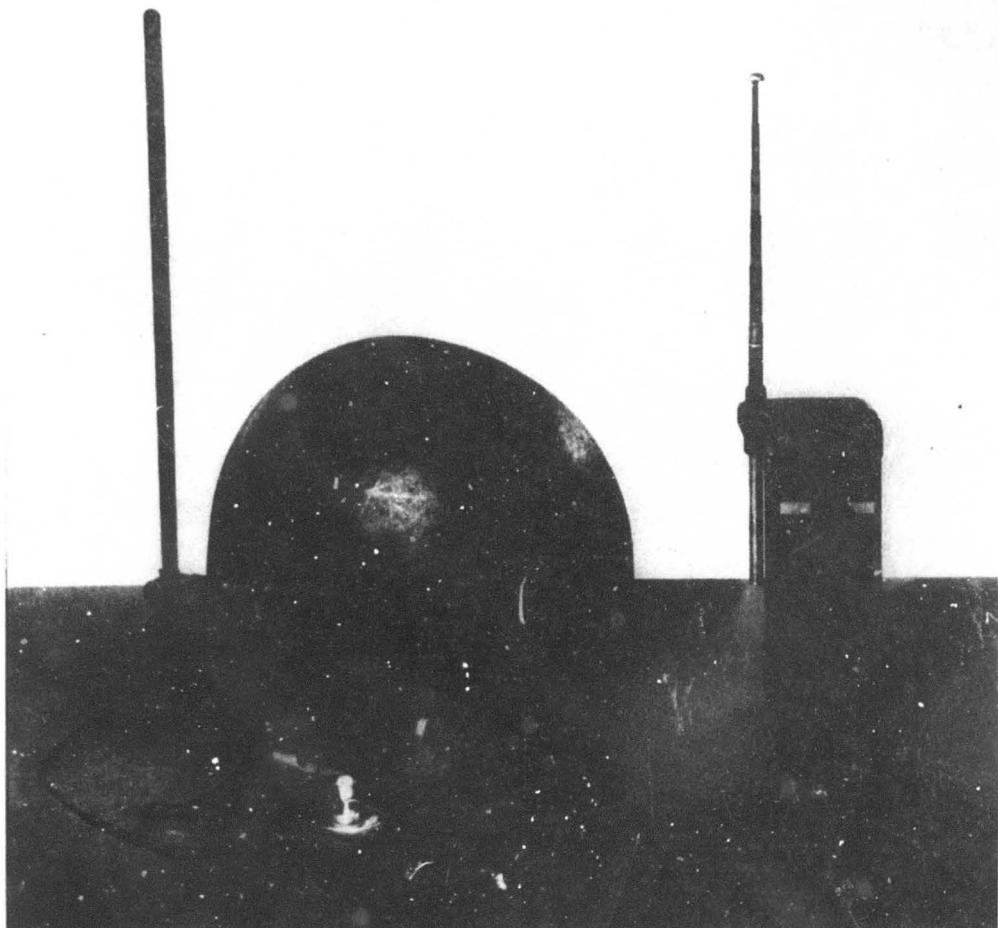
MANPACK EQUIPMENT: This portable equipment is often housed and trans-
ported in a single case. Design consideration must also be given to
exposure to environmental extremes, as well as the transport shock and
vibration inputs.

EXAMPLES OF CLASS I THROUGH VI EQUIPMENT (continued)
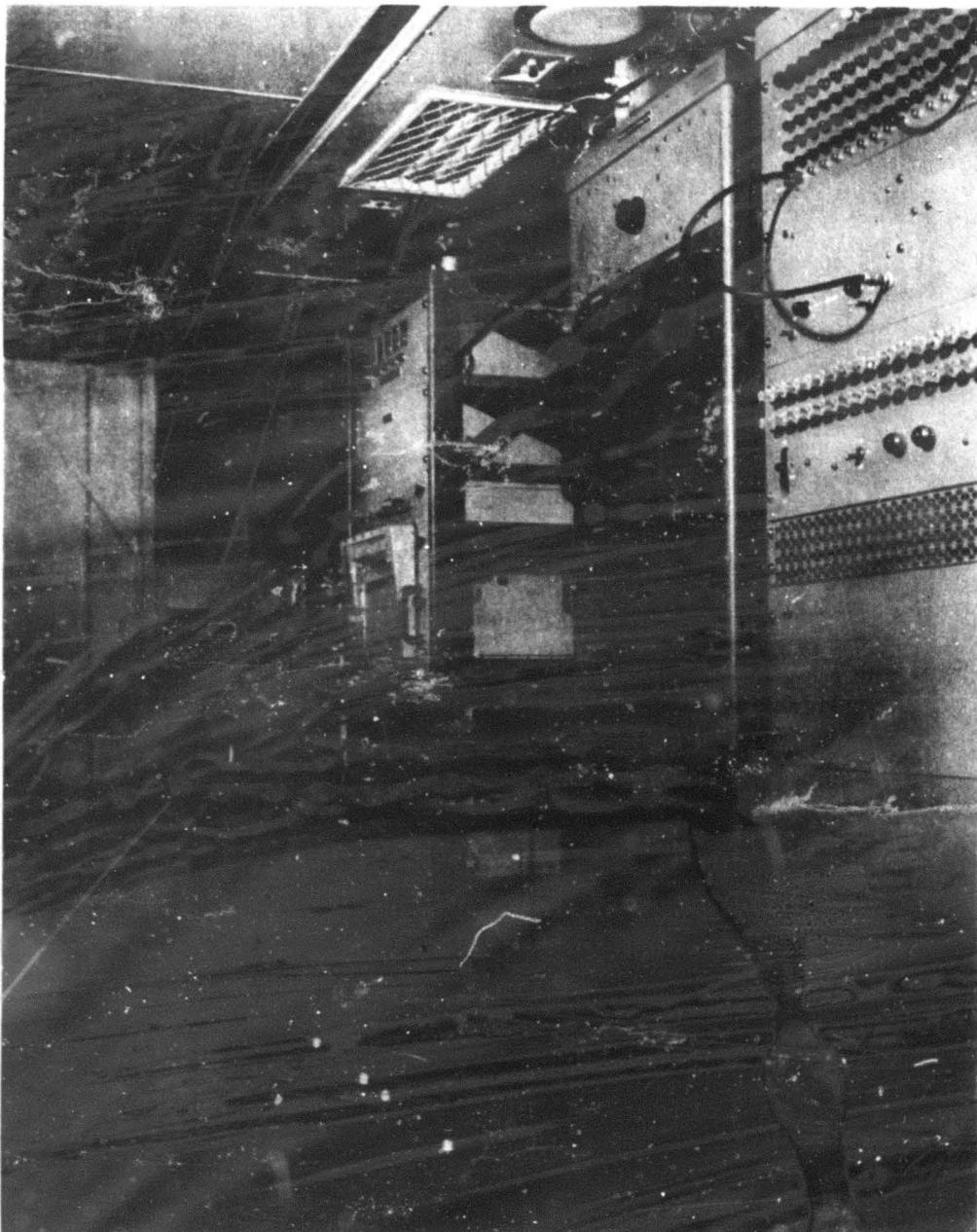
CLASS II EQUIPMENT: This category includes equipment which is normally installed in a shelter or van, and is operated within this shelter while the transport vehicle is at rest. The fragile equipment is sometimes shock-mounted.

CLASS III EQUIPMENT: Equipment in this category is installed in much the same manner as Class II Equipment, except that the equipment systems are required to function while the vehicle is in motion.

EXAMPLES OF CLASS I THROUGH VI EQUIPMENT (continued)
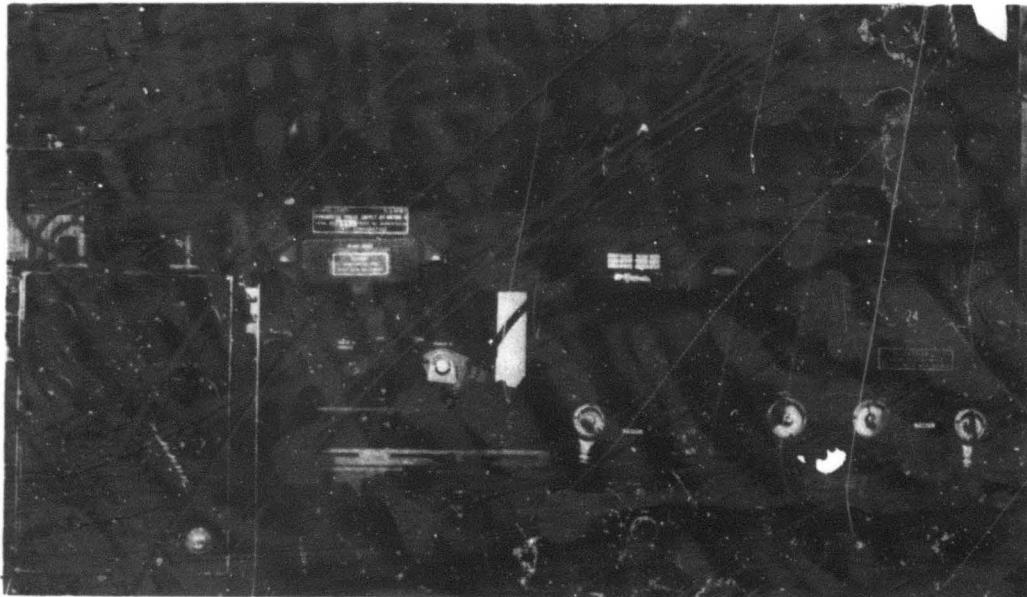
CLASS II AND III EQUIPMENT: Equipment in these classes are firmly
affixed to the transport vehicle. They may or may not be constrained
to operated while the vehicle is in motion.
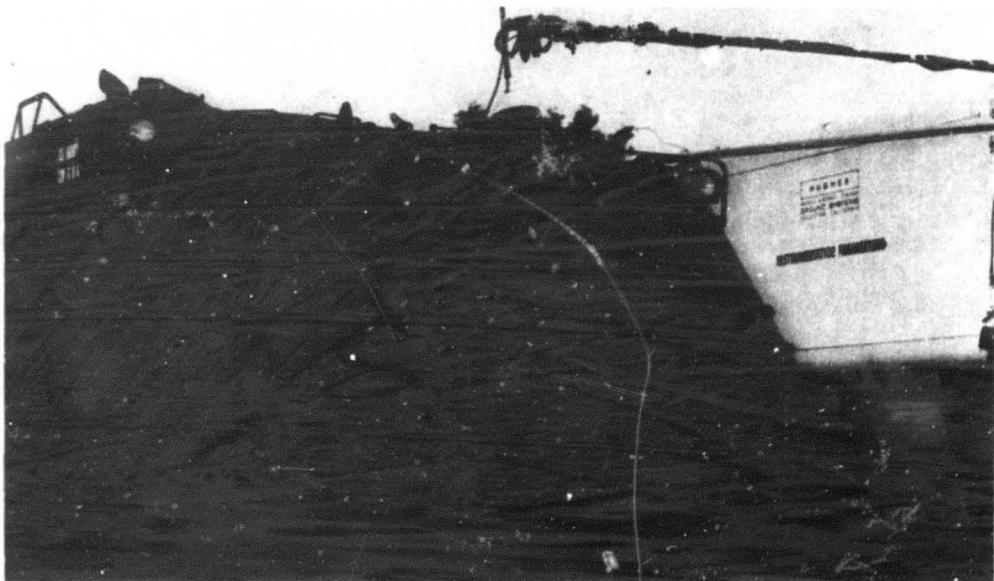
CLASS IV AND V EQUIPMENTS: Equipment in these classes are normally
installed in (firmly affixed to) a tracked vehicle, or an equipment
shelter which is transported by a tracked vehicle. The distinction
between classes IV and V depends upon whether the equipment is con-
strained to operate in motion (V) or at rest (IV).
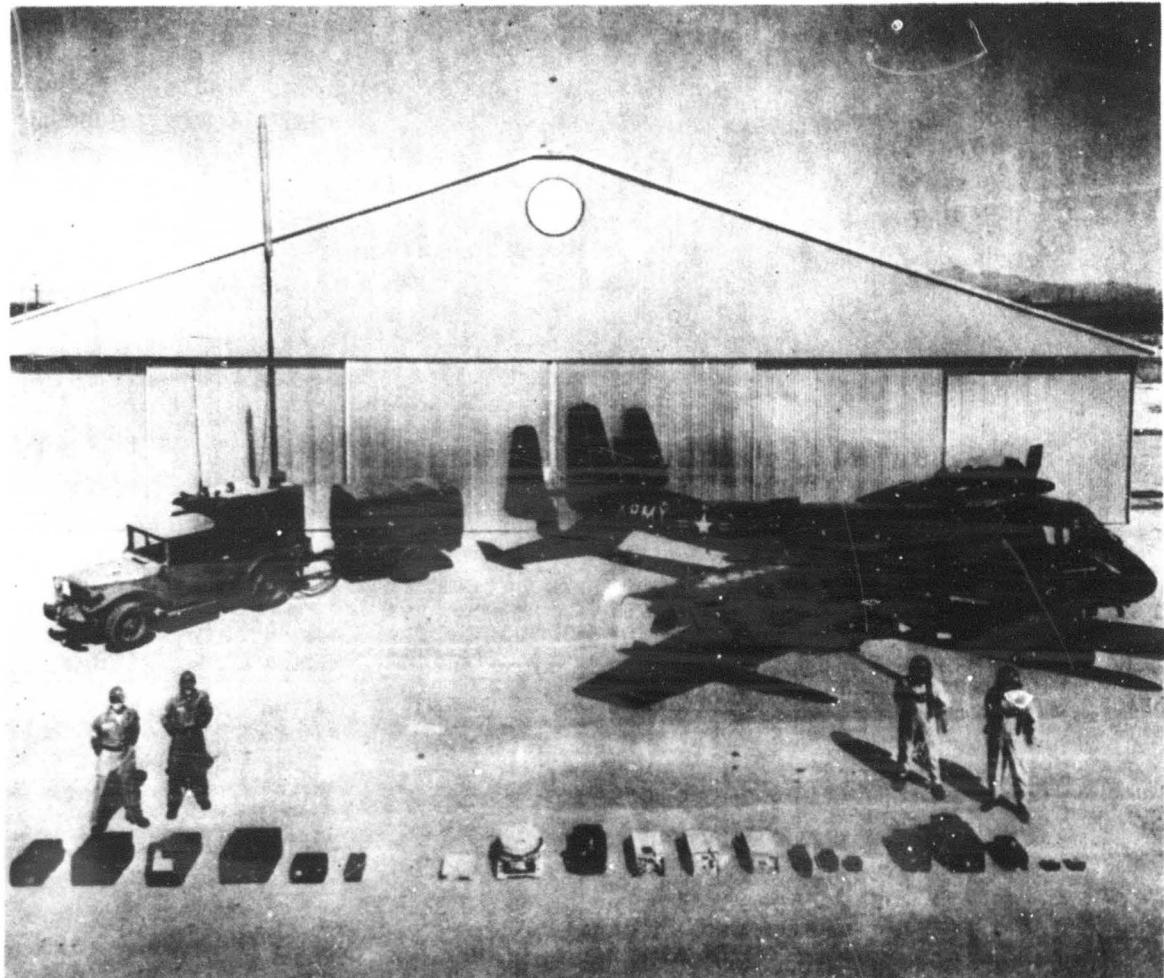
CLASS II/III AND VI EQUIPMENT: Although there may be a functional similarity between equipment classes, the method of transport may greatly affect the structural necessities of the equipment package.

EXAMPLES OF CLASS I THROUGH VI EQUIPMENT (continued)

CLASS VI EQUIPMENT: Army equipment in this class is normally installed in an Army aircraft. These aircraft include a variety of helicopters, short takeoff and landing (STOL) aircraft, and a range of propellor driven observation and transport aircraft. Equipment in this class is normally operational while under stress from the shock and vibration environment.

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MOUNTING AND TRANSPORT ILLUSTRATIONS
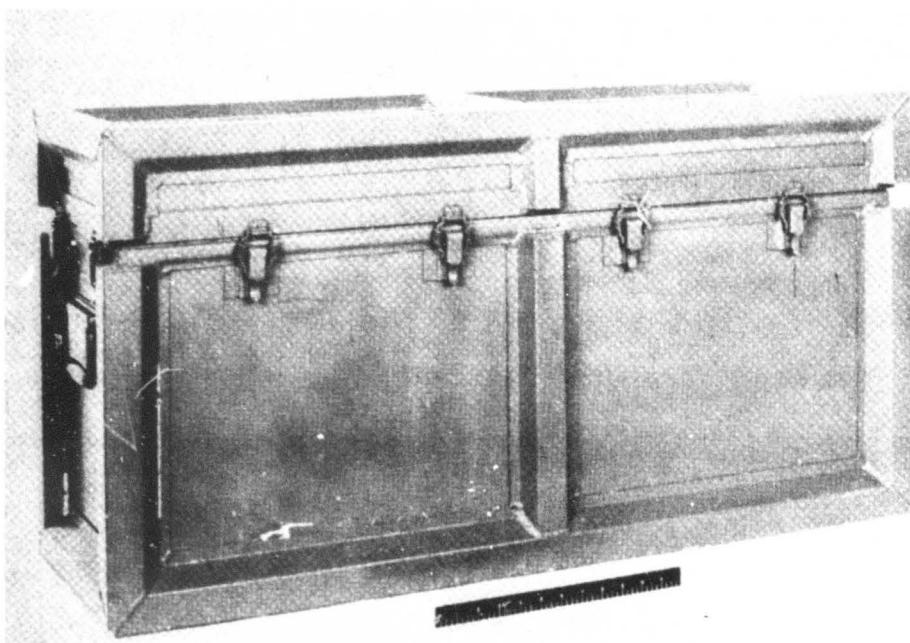
TRANSIT CASE: Class I Equipment is frequently housed in a rugged transit case. This is particularily true of equipment that is to be transported loose in the rear of a vehicle
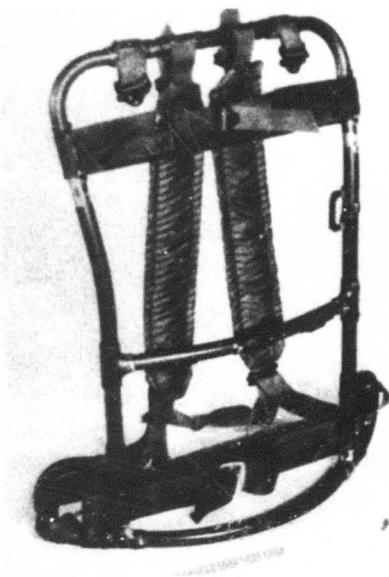
Rucksack
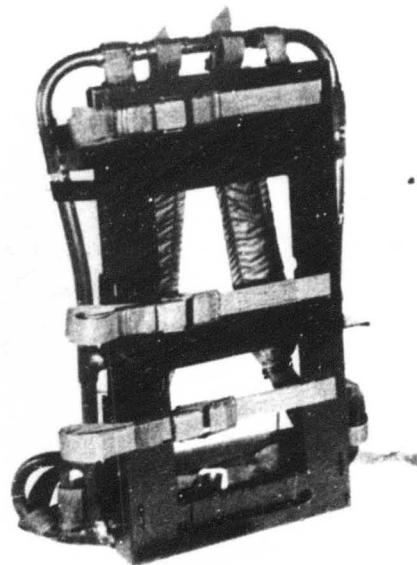
Rucksack and Frame

RUCKSACK: Equipment in Class I is often field transported by manpacking. These equipments are usually highly transportable, and are operated from their transit cases in the raw environment.
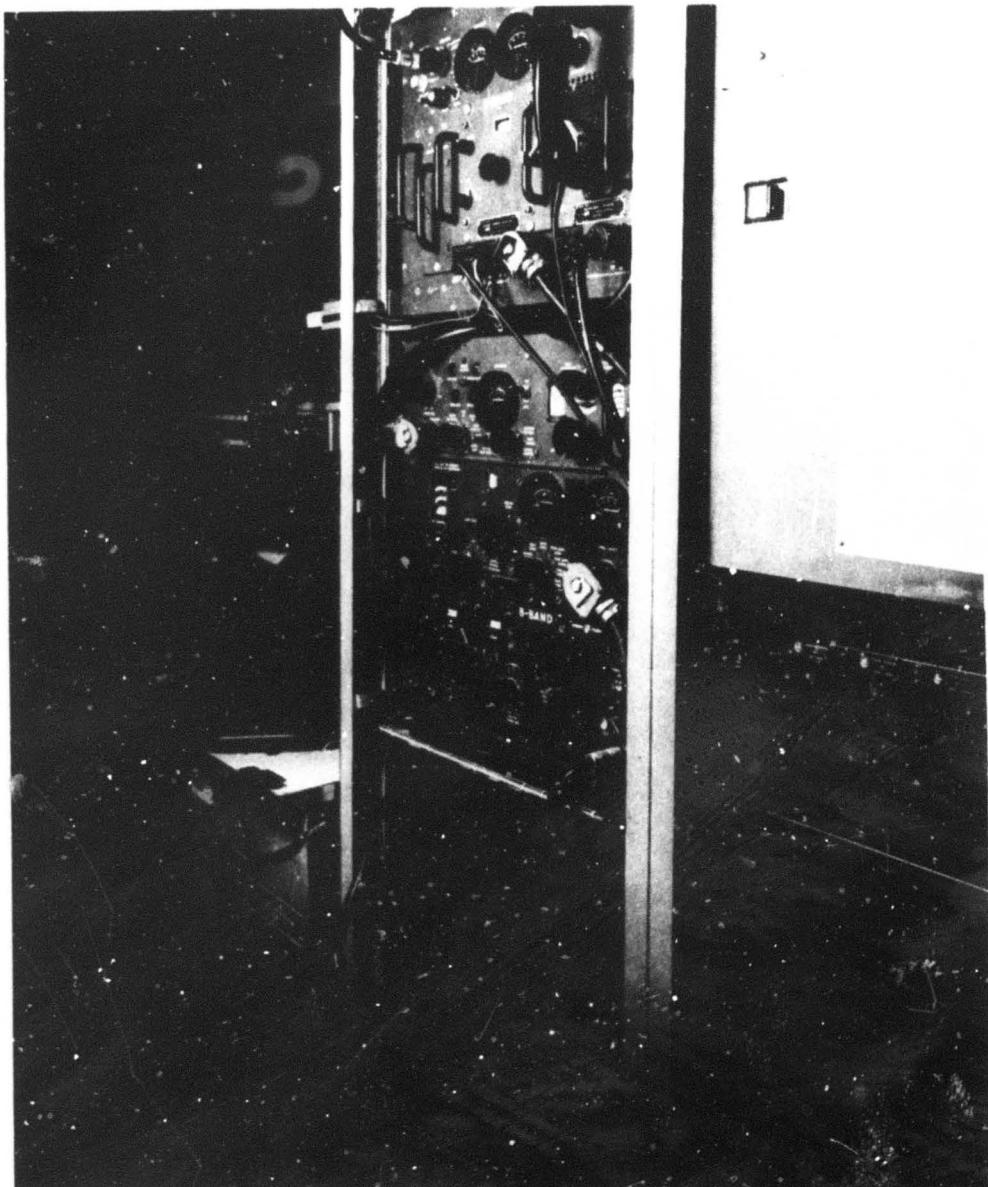
SHELTER MOUNTED EQUIPMENT: Class II Equipment is normally operated while firmly affixed to the vehicular shelter, while the vehicle is at rest. The van or shelter will normally provide some measure of protection from the external environments.

MOUNTING AND TRANSPORT ILLUSTRATIONS (continued)
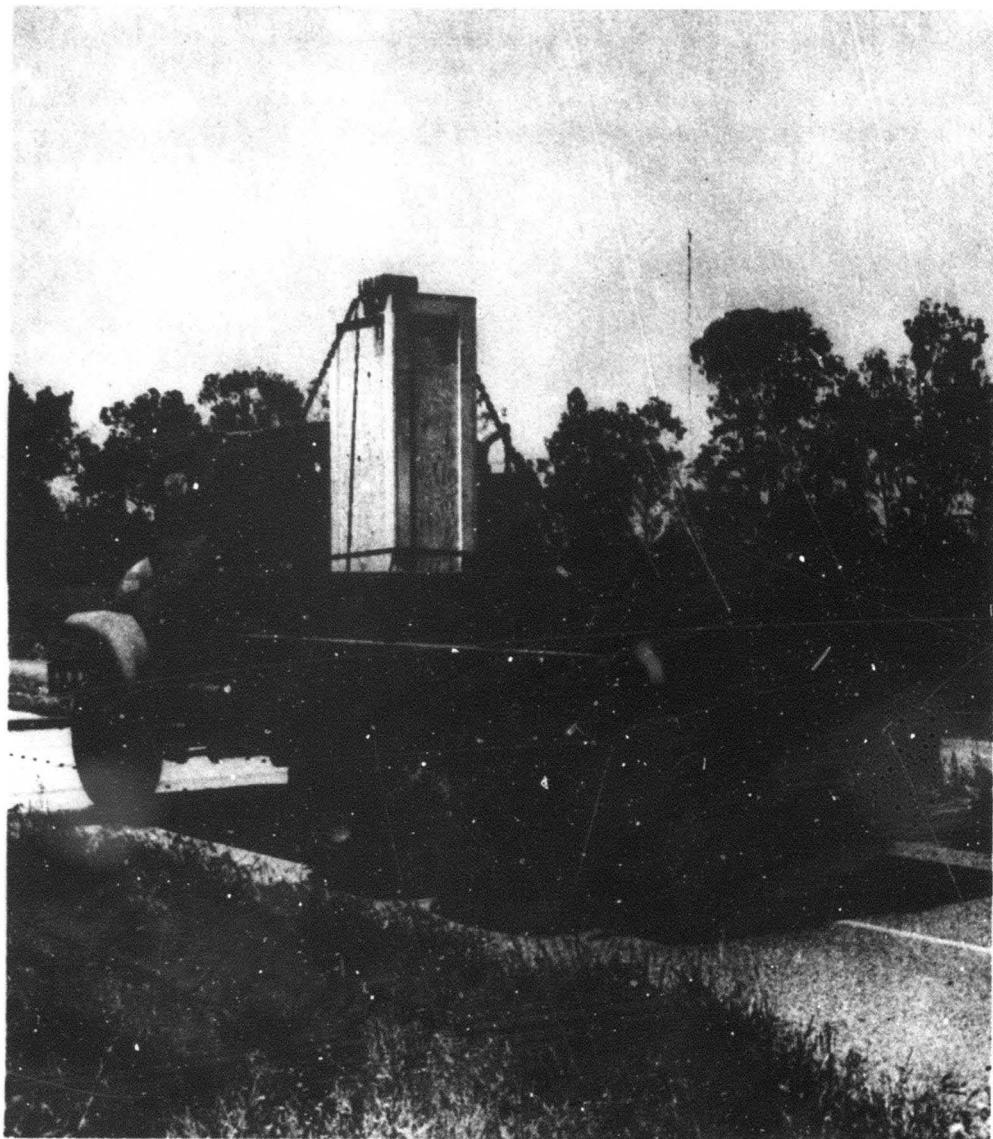
TRANSPORT BY WHEELED VEHICLE: Virtually all classes of Army Equipment are constrained to be transportable by ground vehicle.
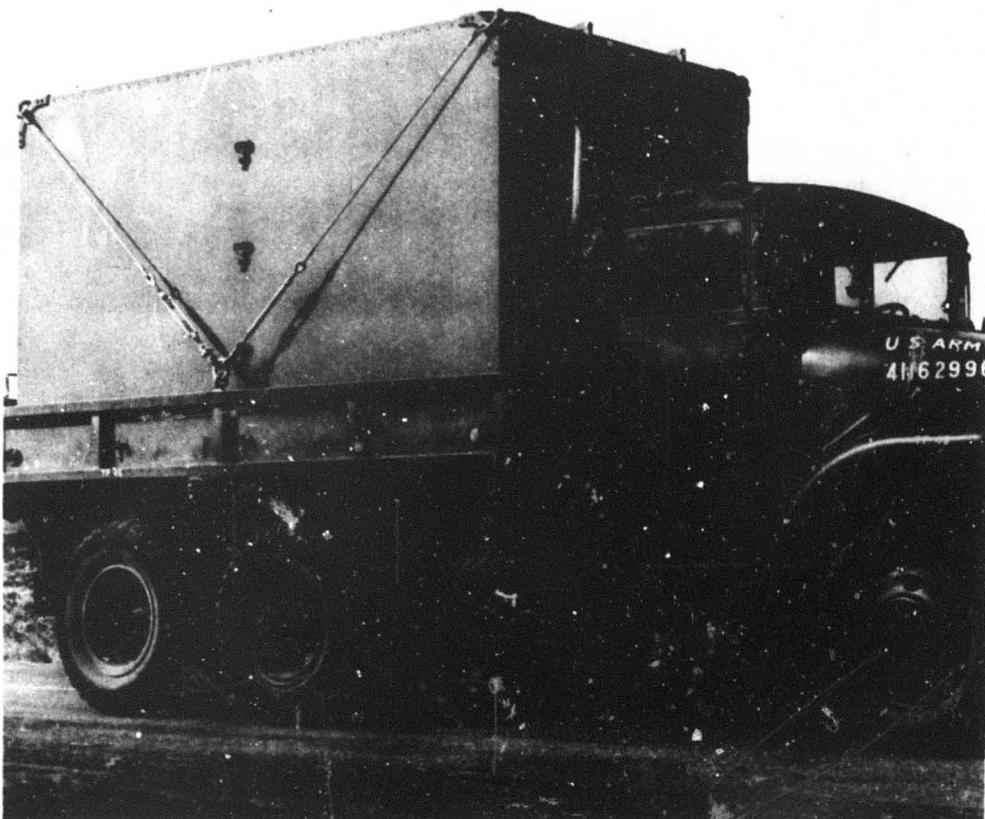
SHELTER MOUNTED EQUIPMENT: These equipments are normally operated
from within the transport shelter and takes full benefit of the pro-
tection afforded by the shelter or van. The shelter or van is
normally affixed to a vehicle while being transported in the field.

MOUNTING AND TRANSPORT ILLUSTRATIONS (continued)
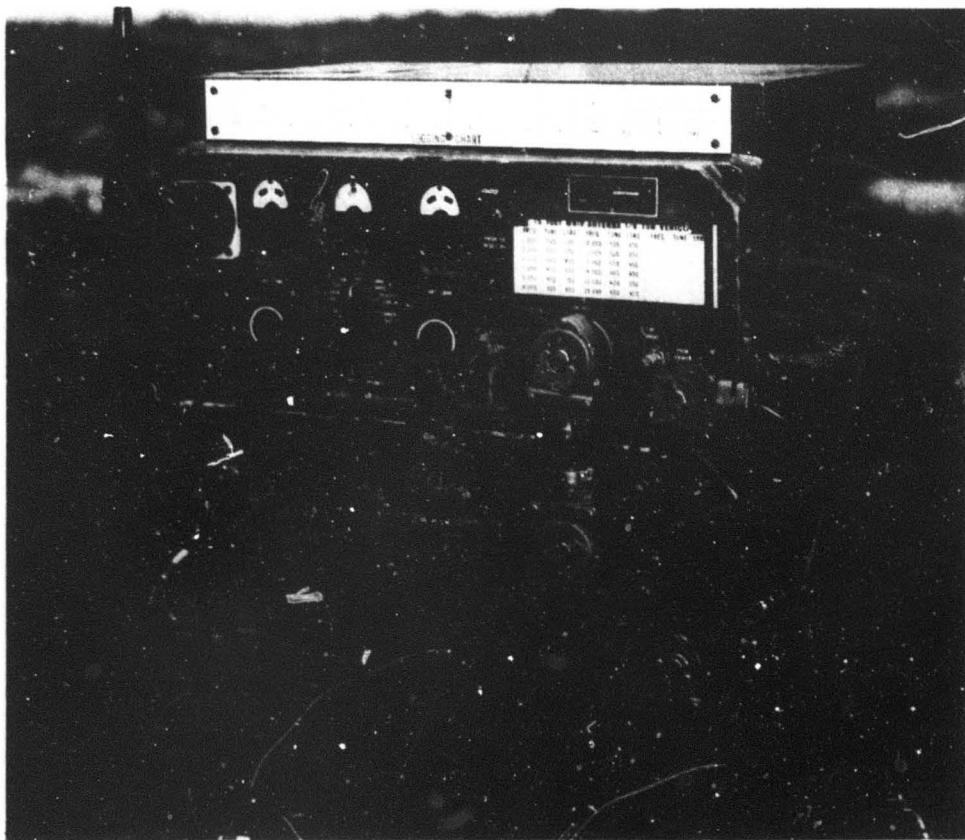
VEHICULAR MOUNTED EQUIPMENT: Equipment in this category is normally
operated when firmly affixed to the transport vehicle. The equipment
may be operative or inoperative while in motion, as constrained by
the equipment class.

AIR TRANSPORTED EQUIPMENT: Virtually all classes of Army equipment
which are not permanently affixed to a vehicle, are required to survive
transport by air. These transport aircraft include; helicopters, both
slung-lifted and cargo area transported; propellor and turbo-prop trans-
port aircraft; and jet powered transport aircraft.

VOLUME II
ANALYTICAL PROCEDURES

VOLUME II
ANALYTICAL PROCEDURES

ABSTRACT:

The objectives of this volume are two-fold. Primarily, its intent is to provide engineers at the design and supervisory levels with specific information and methods for evaluating the structural adequacy of USAECOM equipment. The specific information is provided by presenting the dynamic characteristics of the various quality assurance tests required by USAECOM. The evaluation methods, in addition to satisfying this primary objective, are also involved with a secondary (and perhaps even broader) objective; to provide a unified method of presenting general design requirements. This is accomplished by development of general response spectra, which are the means of translating various dynamic environments into dynamic responses of equipment. Responses are presented in terms of static equivalent acceleration (a force-related quantity which is relevant for structural design) as a function of equipment natural frequency. The methods for generating these spectra are developed for shock, random, and sinusoidal excitations. The methods are based on the separation of basic information into two different factors, one involving the characteristics of the dynamic environment and the other involving the structural characteristics of the item under study. The dynamic environments are determined on the basis of the specific tests involved. The structural data are organized in terms of structural transfer functions for the different types of excitations. The static equivalent accelerations are then derived by operating on the inputs with the appropriate transfer functions.

Volume II - Analytical Procedures

Page	Paragraph	Line	Correction
1-1	Graphic	Center Column	add "Inputs" into "Analytical Evaluation" block
1-4		9	amplification factor
2-3	Graphic	13/14	delete "(V)"
3-1	Graphic	Abcissa	delete "Natural"
3-2	Graphic	25	....may be determined by direct ratio.
3-8	1	9	....the <u>usual</u> transmissibility....
3-8	4	5	The <u>maximum</u> (3-sigma)....
3-9	Graphic	last eqn.	Add ————————— (1)
3-10	1	2	factor <u>of</u> two
3-12	1	14	W <u>pounds</u>
3-12	last eqn.		$f_n = \dfrac{3.13}{\sqrt{.013}}\sqrt{\dfrac{EI}{WL^3}}$
3-13	3	eqn.	demoninator should be $\left| f_n^2 - f^2 \right|$ (absolute value)
3-13	Graphic	eqn. (1)	"m" should be "M"
3-16	2	2	inclu<u>d</u>es
3-16	3	8	delete "(P)"
4-1	Caption	3	modes may, however, be...
4-4	3	5	Several <u>methods</u> are
5-0	4	5	delete "as shown"
5-2	Thesis	2	...applied loads, and is...
6-0	Thesis	1	An illustrative example will...
6-1	Graphic	-	depth dimension is 24"
6-2	2	2	...unit <u>were</u> bolted ...
6-2	5	1	...in the side-side direction...
6-5	2	E eqn	Modul<u>us</u> of Elasticity
6-11	#5	Plot	Ordinate units are "(g)" Delete "Item" Center Frequency is "250 Hz"
6-15	Graphic	R.H. Plot	"Vibration <u>Load</u>"
6-16	4	13	...parallel in <u>the</u>

Volume II - Analytical Procedures

Page	Paragraph	Line	Correction
6-16	5	7	Add "(assumed to be the new f_n)" after 300 Hz.
7-3	2	1-2	Should read "The response acceleration of a system expressed as a number of g's."
7-6	1	12	Add note "(Note that this assumes no deflection in the impacted surface)"
7-6	2	3	Equation is for "$\ddot{X}$"
7-8	3	1	...preceding topic,

VOLUME II
ANALYTICAL PROCEDURES

CONTENTS

CONTENTS (Continued)

VOLUME II

ANALYTICAL PROCEDURES

SECTION 1 — INTRODUCTION

- General Analysis Approach

- Elements of the Analytical Procedure

- Idealising the Complex System

- Limitations in the Approach — Accuracy

- Limitations in the Approach — Resolution

GENERAL ANALYSIS APPROACH

The evaluation of an equipment design is based on the characteristics of the design itself as well as the characteristics of the environments to which it will be exposed.

The analysis approach presented in this volume is intended as a guide toward the evaluation of equipment designed for USAECOM usage. The intended scope of the information can best be defined by first pointing out what the intent is not. It is not meant to teach a project engineer enough to have him replace the technical specialist in the detailed analysis tasks. It is not detailed enough to allow its user to approve all the manufacturing drawings associated with a design. Neither will it provide every information detail needed to describe the high frequency dynamic environments which will be encountered by some of the small instruments and components within an equipment system.

On the other hand, sufficient information is presented to facilitate the preliminary analysis of a design, leading to a decision on its basic strength adequacy. In addition, analytical procedures are presented for evaluating the adequacy of existing equipment and for determining the effectiveness of structural modifications. This approach is in keeping with the basic philosophy of the Design Guide; it is primarily intended for use by the Mechanical Project Engineer and Design Supervisor, although much of the information presented will be of value to the dynamic analyst, structures specialist, and packaging designer.

The general approach to the evaluation of an equipment system as it is applied in this Design Guide involves the determination of a series of discrete, definable, quanta of information: input, transfer function, response, fragility, and margin of safety. These parameters are quantified by analysis as the design concept is established or are measured from the actual equipment or dynamically similar model when available. The inter-relationships of these parameters and how they lead to a value judgment based upon numbers is shown schematically in the adjacent figure.

The analytical procedure offered in this Design Guide is based upon the premise that the complex response of an equipment to a given dynamic excitation may be presented in simplified notation as:

Static Equivalent Acceleration = (Input Excitation) x (Transfer Function)

These factors, as will be developed in later sections of this volume, lead to a forced-related (and therefore design-relevant) load which may be compared to some known level of strength pertaining to the element under scrutiny.

The reason for establishing the two separate factors (input excitation and transfer function) is that this makes it possible to separate the environmental characteristics from the design characteristics. The input excitation will be dictated completely by the equipment "class" (which is determined by its intended usage) and the tests required for each class. The transfer function depends on the dynamic response characteristics of the equipment itself, the form of the transfer function depending on the type of test being considered.

The qualitative and quantitative measures of design adequacy, besides providing value judgments, also serve to indicate the presence of design deficiencies. Thus the means are provided for determining what design changes will improve the dynamic structural integrity of the design. It should be noted that such changes may be desirable for improvement purposes even if the original design is adequate.

After a design change is effected, the entire evaluation process should be repeated because the properties depending on the design will have been modified. Thus, while the input is the same, the transfer function, response, and fragility will be changed. It is seen, therefore, that the process must be iterated until a satisfactory design is obtained.
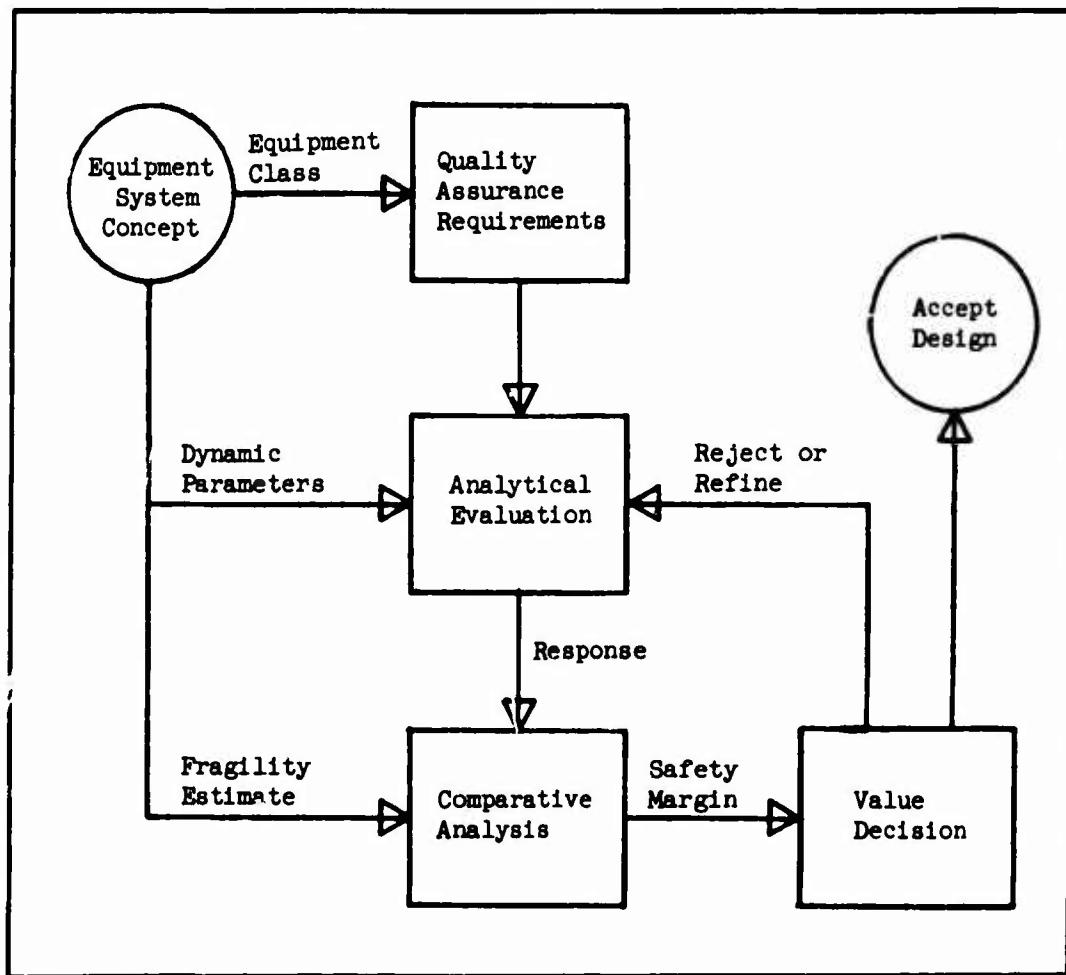
GENERAL ANALYSIS APPROACH: The technical logic of the analysis may be represented by three analytical procedures, leading to a value decision.

ELEMENTS OF THE ANALYTICAL PROCEDURE

The independent variable in the logic chain describing the dynamic excitations unique to Army equipment is equipment "class". The class designation (based on intended usage) leads to a description of the anticipated loadings which can then be compared with fragility.

The operational environment which is anticipated for each unique category or class of Army equipment is simulated by a series of environmental tests or Quality Assurance provisions. The dynamic tests of interest here are defined by the equipment class designation, leading to a set of input excitations used to calculate equipment response. The equipment classes, based on anticipated usage, are summarized in the accompanying table.

The input excitations result from the dynamic environments to which the equipment will be subjected during Quality Assurance testing. The required tests are determined by the equipment class to which the system belongs as illustrated in summary form in the adjacent figure. Each of the tests may be described in terms of the "type" of dynamic loads imposed on the tested equipment. For example, the steady state (or sinusoidal) tests are described by vibration levels as a function of frequency. The adjacent figure summarizes the individual test requirements in terms of this excitation type, a categorization which will be utilized throughout this analysis procedure.

The ultimate objective is to develop curves of G_{se} (equivalent static acceleration) vs f_n (natural frequency of the equipment) for each class of equipment. Thus the responses to each test will be expressed in terms of G_{se} according to the prescribed procedures. The results will then be summarized and arranged by class.

The transfer function is a characteristic of the design itself, being determined by the equipment's natural frequency and maximum dynamic amplification factor. It is used to compute the dynamic responses resulting from the Quality Assurance test dynamic inputs.

The response to the dynamic inputs is expressed in terms of static equivalent acceleration. Thus the response can be used to compute stresses in the structure of the equipment. The response is computed by operating mathematically on the input using the transfer function in a prescribed fashion.

An item's fragility is a quantitative measure of its strength. It is expressed in terms of the maximum dynamic response which the item is capable of surviving. Thus, evaluation of the adequacy of a design can be performed by simply comparing the response and fragility.

Margin of safety is the classical measure of an item's strength. It is a direct comparison of actual stress (or load) with allowable stress (or load), and indicates not only whether a design is adequate, but also how close the design is to being inadequate.

Summary of Equipment Classes

Class I   - Loose cargo and/or manpack.
Class II   - Installed in unarmored vehicle, not operating in motion.
Class III - Installed in unarmored vehicle, operating in motion.
Class IV   - Installed in tracked vehicle, not operating in motion.
Class V   - Installed in tracked vehicle, operating in motion.
Class VI   - Installed in aircraft.

Summary of Quality Assurance Tests

	Class of Equipment					
	I	II	III	IV	V	VI
Vibration, Survey & Dwell (V) (5-500 Hz)						X
Vibration, Survey (V) (10-55 Hz)	X	X		X		
Bounce, Loose Cargo (R)	X					
Bounce, Vehicular (R)		X	X	X	X	
Shock, Ballistic (S)				X	X	
Shock, Bench Handling (S)	X	X	X	X	X	
Shock, Shipping Drop (S)	X	X	X	X	X	
Shock, Crash Safety (S)						X
Road Mobility (R)		X	X	X	X	
Railroad Hump (S)		X		X		

Excitation Types

Vibration (V) (G vs. Frequency)	Random (R) (G^2/Hz vs. Frequency)	Shock (S) (G_{se} vs. Frequency)
Resonance Survey	Bounce, Loose Cargo	Shock, Ballistic
	Bounce, Vehicular	Shock, Bench Handling
Resonance Dwell		Shock, Shipping Drop
	Road Mobility	Shock, Crash Safety
		Railroad Hump

FUNDAMENTAL INFORMATION: The analytical procedures are predicated on the Quality Assurance tests imposed on the various classes of equipment.

IDEALIZING THE COMPLEX SYSTEM

It is always desirable, and usually possible, to idealize even a very complex system
as a simplified model for purposes of evaluating the design.

Most items of equipment encountered under normal circumstances are quite
complex in their dynamic behavior. Any particular item, if it were sub-
jected to a sinusoidal vibration input of constant amplitude and with a
frequency varying from very low to very high, would exhibit many resonant
frequencies. Each such frequency would have an associated mode shape,
with various portions of the item moving in a variety of directions. For
each such frequency there would also be a corresponding maximum response
at some point in the equipment, thus defining the local dynamic amplifi-
cations factor. In order to describe the dynamic behavior of such a
complex system, an analytical model must be generated wherein all the
modes of interest can be described by specific geometric coordinates.
Generally such a model is described in terms of discrete masses and
springs, such that the number of degrees of freedom is at least as great
as the number of resonant frequencies. Obviously, such a task is a very
difficult one.

For purposes of evaluating the major structural elements of a piece of
equipment, it is usually sufficient to consider only the fundamental
(that is, lowest frequency) mode of vibration. This is true because in
the fundamental mode all the elements of the equipment are moving in
phase with each other, thereby producing inertia loads all in the same
direction at any given moment. Thus maximum stresses result in the
major structural elements; this condition is usually the most critical
one for structural design. Limitations and qualifications of this con-
cept are presented in detail in the following topic sections.

If only the fundamental resonant frequency is of interest (as just
described) then it is possible to study the complex system by means of a
simplified dynamic model. For purposes of design evaluation, the complex
system can be idealized as a simple one-degree-of-freedom system. This
consists of three elements: one mass, one spring, and one damper.

The dynamic characteristics of the one-degree-of-freedom system can be
described in terms of resonant frequency (f_n) and maximum dynamic ampli-
fication factor (Q). Knowing these quantities, it is possible to sketch
the applicable response curve, as shown.
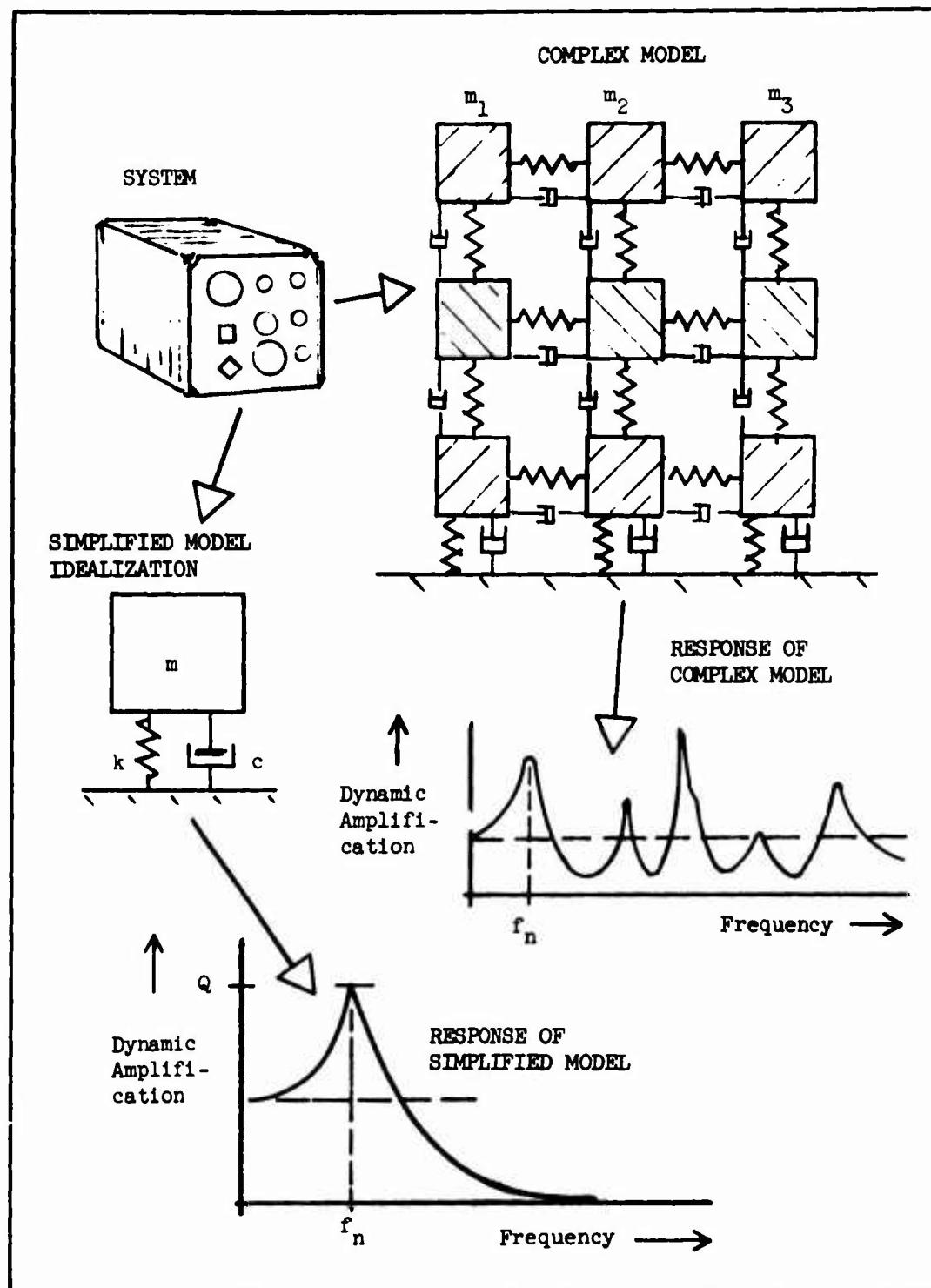
ANALYTICAL IDEALIZATION: A simplified Model is used to determine the dynamic response of a complex system.

LIMITATIONS IN THE APPROACH — ACCURACY

The analysis approach described herein has certain inherent limitations which must be recognized — one of these is accuracy.

A simple structural system can be analyzed quite accurately after the dynamic inputs have been described. This follows because the dynamic behavior of a simple system can be computed readily. On the other hand, the complex systems under discussion here cannot be analyzed quite so accurately. Rather, the analysis approach described herein has certain inherent limitations which must be recognized in order to prevent improper conclusions. The accuracy limitation is discussed below.

The quality assurance tests imposed on USAECOM equipment are sometimes so complex that their dynamic characteristics cannot be described in determinate mathematical form. For example, the bounce test consists of resting the item on a vibrating platform without tying it down. The item bounces in an unpredictable fashion, so that its motions cannot be completely described mathematically. Thus even the starting point in the evaluation of a design — the dynamic inputs — may be obscure. This situation is remedied by specifying the environment in terms of typical values which have been measured during test. Thus the equipment in question is evaluated on the basis of typical, not exact, dynamic inputs.

The idealization of a complex structural item as a one-degree-of-freedom system cannot always be accomplished meaningfully. In certain cases where stiffnesses and/or masses are distributed in a very non-uniform or non-symmetrical fashion, the fundamental mode of vibration may exhibit some masses moving in different directions. In such cases, idealization as a one-degree-of-freedom system will not lead to suitable descriptions of internal stresses. More detail is required in the idealization, which may have to include more degrees of freedom in order to account for the complicated motions.

Whenever it appears that the one-degree-of-freedom idealization is inadequate, an expert in engineering mechanics should be consulted. The significant factors to consider when deciding whether to seek expert advice are unusual stiffness and/or mass distributions. Unusual stiffness comprises those cases in which forces on the structure in a given direction produce deflections in some different direction. Poor mass distributions include those cases in which large masses are concentrated at points remote from the overall center of gravity of the system.

Actual
Response to
Actual Inputs

RESPONSE OF A TYPICAL
MULTI-DEGREE-OF-FREEDOM
SYSTEM

Frequency

Computed
Response to
Idealized
Inputs

RESPONSE OF A ONE-
DEGREE-OF-FREEDOM
SYSTEM

f_n

Frequency
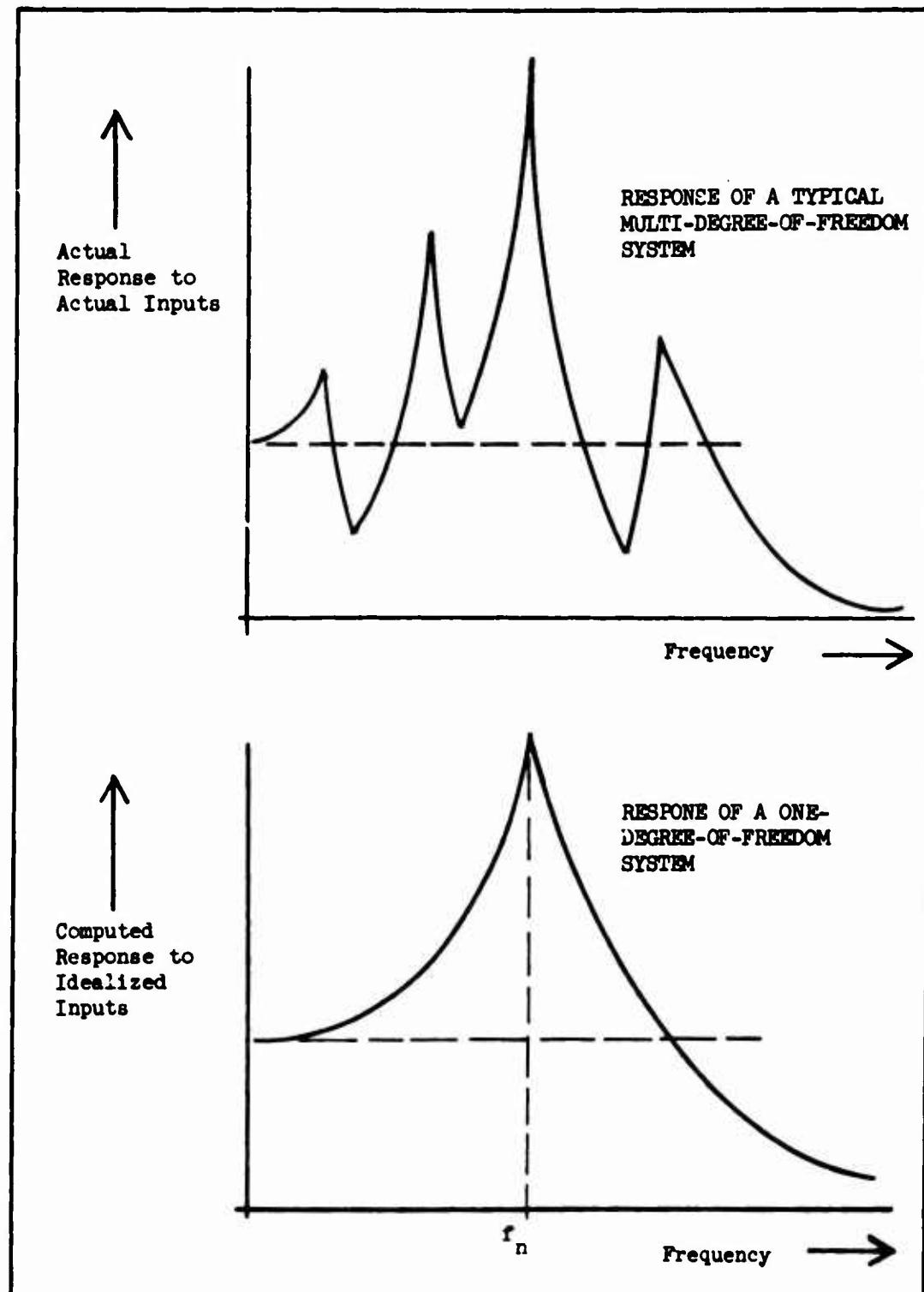
ACCURACY LIMITATION: The idealized response, based on a one-degree-of-freedom model, is useful even though it may not be exact.

LIMITATIONS IN THE APPROACH - RESOLUTION

**The one-degree-of-freedom idealization limits the resolution of the analysis -
small components are ignored.**

The approach taken here treats only the major structural elements of the
system, while ignoring smaller components. This limitation arises from
the one-degree-of-freedom idealization. The result is that if some small
structural element is more likely to fail than the major structure, such
failure will not be predicted. This situation can usually be anticipated
by means of careful qualitative analysis of the system in its entirety
before establishing the idealized analytical model.

Most pieces of equipment are built up from many smaller components, all
interconnected by major and/or minor structural elements. This leads to
a situation in which the first failure to occur in the system might be a
functional failure of a component rather than a structural failure.
Dynamic loads imposed on such components depend on the relationship
between their natural frequencies and the fundamental natural frequency
of the system. For purposes of illustration, consider a two-degree-of-
freedom system consisting of a large mass (representing the major struc-
ture) on which is mounted a small mass (representing the small component)
as shown.

The shock response of the large mass can be computed by classical means,
and is relatively independent of damping. The response is, at most,
twice the input. However, if damping is small, the large mass will
vibrate through many cycles at the shock response level. This will look
like a vibration input to the component, and will be amplified according
to the amount of damping associated with the component. Thus the com-
ponent response can be many times as great as the original shock input.
This situation may arise whenever the natural frequency of the component
is close to the fundamental natural frequency of the system.

In the event that the component has a high natural frequency relative to
that of the system, the response to vibratory inputs could be large at
high frequencies. The conditions giving rise to this situation are high
damping in the structure (and therefore high transmission above resonance)
and low damping in the component (and therefore high amplification at
resonance). This situation is illustrated in the accompanying diagram.

It is seen that, although the one-degree-of-freedom approach is very use-
ful, there are certain limitations which must be kept in mind. It is
advisable that an expert in the field of engineering mechanics be con-
sulted whenever it appears that either of the following situations may
develop: the natural frequency of a small component is close to the fun-
damental natural frequency of the system; the major structure is highly
damped and some high-frequency component is lightly damped.
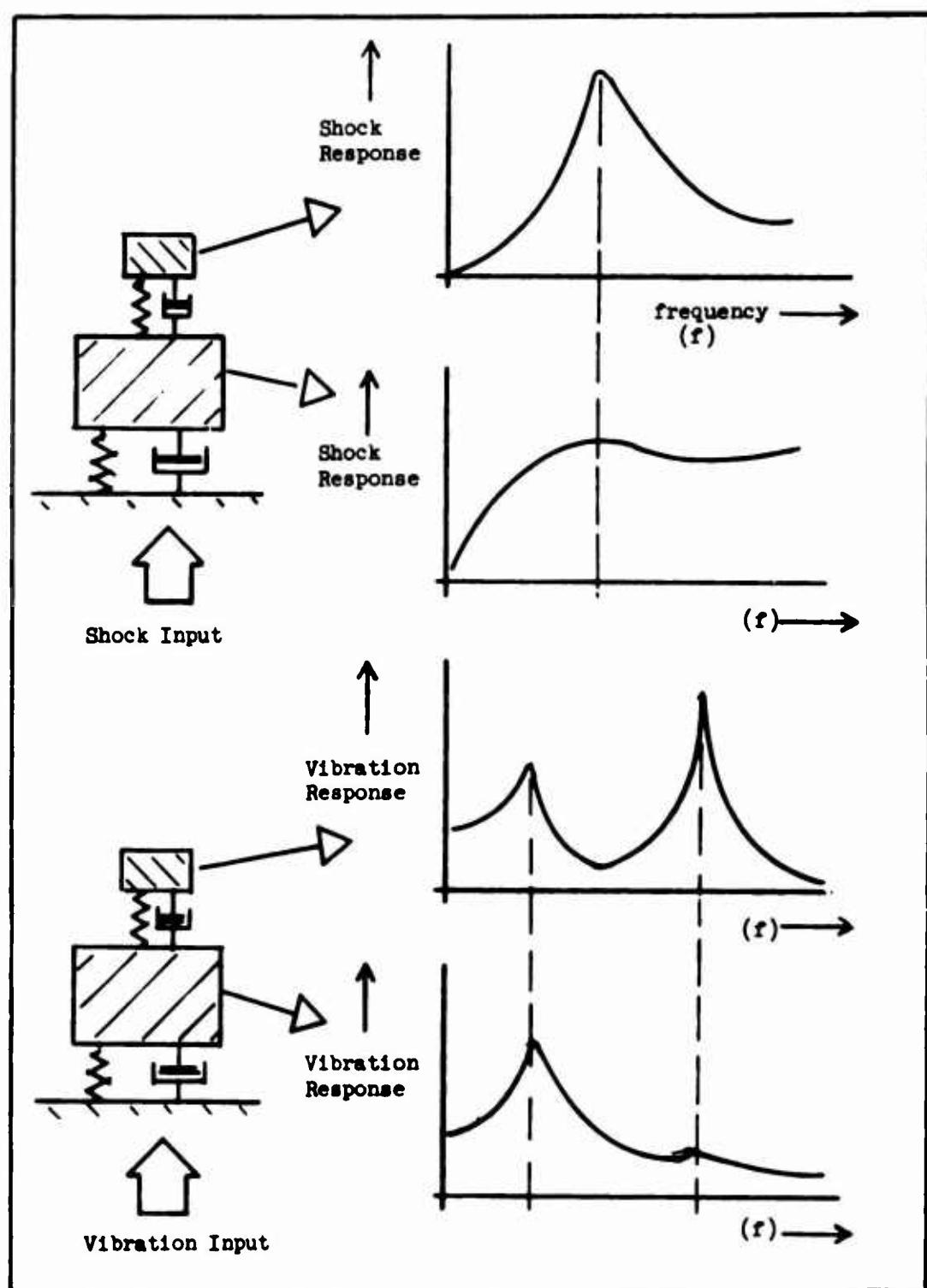
RESOLUTION LIMITATION: The response of small items is not considered in the analysis due to the one-degree-of-freedom idealization.

VOLUME II

ANALYTICAL PROCEDURES

SECTION 2 — SHOCK AND VIBRATION INPUTS

- Inputs and Quality Assurance Provisions

- Equipment Classes and Quality Assurance Tests

- Input Excitations in the Frequency Domain - Class I-VI

INPUTS AND QUALITY ASSURANCE PROVISIONS

The shock and vibration inputs to an item of equipment depend on the quality assurance tests to which it must be subjected. These tests are meant to simulate actual service environments.

Equipment supplied for use by the United States Army may be subjected to any number of different environments, ranging from a quiet office to explosion exposure during battle. If the actual environment of a particular item of equipment were known, then the item could be designed and tested to that specific environment. This can sometimes be accomplished for certain special equipment. In the main, however, any particular item of equipment could be exposed to a variety of environments. The uncertainty is alleviated to some extent by identifying various classes of equipment, wherein the classifications are made on the basis of anticipated severity of service. The particular classes were defined in the Introduction, Section 1. These classes encompass all types of equipment purchased by the United States Army. Therefore they can serve as guidelines for classification of environments.

In order to provide some assurance that the equipment will perform satisfactorily in service, certain quality assurance tests have been specified by the Army. These tests are designed on the basis of the actual environmental dynamics, and are specified according to the aforementioned class categorization. The quality assurance tests are only representative, in the sense that they may be either too severe or not severe enough relative to the actual environment experienced by a particular piece of equipment. Thus the fact that a particular design passes (or fails) the required testing does not guarantee that each item will never (or always) fail in actual service.

For purposes of design and analysis, the tests have been described in terms of their dynamic characteristics. That is, each test produces a certain dynamic input into the tested item. These inputs have been determined so that they can be catalogued by class and by test.

The particular tests required are shown in the following topic. They can be grouped into categories reflecting the nature of the mathematical description of their inputs to the equipment to be tested. These categories are shock, sinusoidal vibration, and random excitation. This categorization will make it possible to compute equipment responses to the tests by means of standard analytical methods easily described and easily utilized. The categories are denoted in the table of tests* by the letters in parentheses: "S" refers to shock, "V" refers to sinusoidal vibration, "R" refers to random excitation. Also included in the following topic is a repetition of the class descriptions. It is repeated there for convenience and easy reference for comparison with the table of tests.

---

*Table shown in the following topic.

TYPICAL SERVICE TRANSPORT AND
MOBILIZING VEHICLES

Ground

- Rail Flatcar (open)
- Rail Boxcar (enclosed)
- Wheeled Vehicles
- Tracked Vehicles
- Trailer Complexes

Sea

- Cargo Ship
- Amphibious Carrier

Air

- Jet Cargo Aircraft
- Propeller Cargo Aircraft
- Helicopter

RANGE OF REQUIRED QUALITY
ASSURANCE TESTS

- Sinusoidal Vibration
- Drop Shock
- Ballistic Shock
- Rail Humping
- Bounce
- Road Mobility

Static
Equivalent
Acceleration

ENVELOPES
ARRANGED BY
EQUIPMENT CLASS

Frequency ⟶

QUALITY ASSURANCE TESTS: The range of dynamic tests which are used to
qualify a given equipment system, reflect the transport needs of the
equipment class.

EQUIPMENT CLASSES AND QUALITY ASSURANCE TESTS

The detail definitions of equipment class serve to define the quality assurance tests required for any particular item of equipment.

1. <u>Classes of equipment</u> — Equipments shall be considered to be of the following classes, according to their basic use and mode of transportation employed in that use:

  a. <u>Class I</u>. Includes equipment which will be fie.. transported as loose cargo and/or manpack. Equipment designed in a combination shipping and operation container or transit case is included in this class. The equipment shall be capable of being shipped by rail, truck, sea, or air.

  b. <u>Class II</u>. Includes equipment which is installed in (firmly affixed to) an unarmored vehicle, shelter, or van and is operated when the vehicle, shelter, or van is not in motion. The installed equipment shall be capable of being shipped by rail, truck, sea, or air.

  c. <u>Class III</u>. Includes equipment installed the same way as in Class II and is operated while the vehicle, shelter, or van is in motion.

  d. <u>Class IV</u>. Includes equipment which is installed in (firmly affixed to) a tracked vehicle, or shelter, or van mounted to a tracked vehicle, and is operated when the vehicle is not in motion. This vehicle may be either armored or unarmored. The installed equipment shall be capable of being shipped by rail, truck, sea, or air.

  e. <u>Class V</u>. Includes equipment installed the same way as in Class IV and is operated while the vehicle, shelter, or van is in motion.

  f. <u>Class VI</u>. Includes equipment installed in or transported by aircraft. The equipment may be operated in flight in cases of aircraft installed systems. The air transported equipment is not normally operated while airborne.

2. <u>Definition of equipments</u>

  a. <u>Manpack equipment</u>. Equipment in this category is operated in the field without benefit of vehicle, shelter, or van protection. Equipment is normally transported loose in the rear of a vehicle, but operated external to the vehicle. Equipment is portable, and may be housed in a combination of transit case and equipment package.

  b. <u>Vehicular mounted equipment</u>. Equipment in this category is normally operated when firmly affixed to a vehicle, either with or without shock mounts. Vehicle will provide a limited amount of protection from the external environments.

  c. <u>Shelter or van mounted equipment</u>. Equipment in this category is normally operated from within and takes full benefit of the protection afforded by shelter or van. The shelter or van is normally affixed to a vehicle when being transported in the field.

d. <u>Airborne equipment</u>. Equipment in this category is normally operational while airborne in the case of installed equipments. The equipment will thus be subject to the range of environments associated with these aircraft. Equipment that is transported only by air will be subjected to the environment of the cargo area of helicopters, propeller aircraft, and jet aircraft while in the non-operational condition.

Test To Be Performed	Class of Equipment					
Equipment Operating	I	II	III	IV	V	VI
Vibration (Resonance Search 10-55 Hz)(V)	X*					
Vibration (5-500 Hz)(V)						X
Vibration (Resonance Dwell)(V)						X
Bounce, Loose Cargo (R)	X*					
Bounce, Vehicular (R)			X		X	
Shock, Ballistic (S)					X	
Munson Road Course (R)			X		X	
Perrimen Road Course (R)			X		X	
Equipment Non-operating						
Vibration (Resonance Search 10-55 Hz)(V)	X	X		X		
Bounce, Loose Cargo (V)(R)	X					
Bounce, Vehicular (V)(R)		X		X		
Shock, Ballistic (S)				X		
Shock, Bench Handling (S)	X	X	X	X	X	
Shock, Drop (S)	X	X	X	X	X	
Shock, Crash Safety (S)						X
Munson Road Course (R)		X		X		
Perrimen Road Course (R)		X		X		
Railroad Hump Test (S)		X		X		

*In special cases only.

QUALITY ASSURANCE TESTS: Illustrated are the matrix of tests normally constrained to Army equipment, arranged by equipment class.

INPUT EXCITATIONS IN THE FREQUENCY DOMAIN

The quality assurance tests can be described in terms of the dynamic inputs seen by the tested equipment.

Each dynamic test can be described by its dynamic characteristics. In the case of the USAECOM tests, the tests are categorized according to the nature of these excitations: shock, sinusoidal, and random. In general, if sufficient information were available, each shock test could be described in terms of the acceleration pulse applied to the equipment. This is the case when a shock test is specified in the form of a shaped pulse with given values for pulse amplitude, shape, and duration. However, such a concise description is not available for the shock tests anticipated here — ballistic shocks and drop tests. Here, where the input is not known and is practically impossible to determine, the test characteristics are described in terms of the response of the tested item. Data are available which correlate the maximum response of the tested item (assumed to be a one-degree-of-freedom system) with its resonant frequency. These data can be presented in graphical form as a plot of maximum response acceleration versus system resonant frequency. Such a graph is known as a "shock spectrum." Sinusoidal test parameters are generally known in terms of input acceleration (or displacement) as a function of excitation frequency, for these are the parameters usually specified. Tests which appear to produce random inputs into the tested equipment can be described in terms of power spectral density, representing the level of excitation energy present as a function of frequency. Such a description is sometimes necessary even though the test is not theoretically random (non-deterministic) in nature, because the actual input is so complicated as to be impossible to describe otherwise. This is the case for the road tests and cargo bounce tests. The various tests, arranged in the aforementioned three categories, are described below.

Included under the heading of shock tests are the ballistic shock, the railroad hump tests, and the bench handling, drop, and crash safety tests. Shock response spectra are available for each of these tests. The sources of the information are:

Ballistic Shock:	Volume III, Chapter 6 of the Design Guide
Railroad Hump:	Volume III, Chapter 6 of the Design Guide
Bench Handling:	Appendix
Drop Test:	Appendix
Crash Safety:	MIL-STD-810

The individual shock spectra for these tests are presented in later topics in this section.

The drop tests required for USAECOM equipment comprise several different drop heights for different weights of equipment. The heights specified are 12, 24, 30, 36, and 48 inches. The shock spectra illustrated later correspond to the worst case — that is, the maximum drop height only. Since the shock response level is proportional to the square root of height (see Appendix), the shock spectra for heights other than 48 inches can be determined directly from those given.

Sinusoidal tests number only three: resonance search, vibration sweeps
from 5 to 500 Hz, and dwells at resonance. The resonance searches are
performed at low levels and so are not critical with respect to strength.
The resonance dwells are performed at the same levels as the frequency
sweeps, so they will not be singled out. The input vibration levels are
shown in the following diagrams and topics.

The tests characterized by random excitations are the bounce and road
tests. Available test data for the bounce tests are presented in the
following graphs. Also shown are the excitations for the Munson Road
Course. All these random excitations are in the form of power spectral
densities as functions of frequency.

It should be noted that these test inputs are not independent of the
tested item, but rather the dynamic impedance of the item will have an
influence on the resulting motion. All the test data used to develop
the accompanying inputs were generated with nominal loads on the test
machinery. Thus they should be valid and, in most instances, relatively
insensitive to changes in the tested item.

INPUT EXCITATIONS FOR EQUIPMENT CLASS I

SINUSOIDAL VIBRATION - Not Applicable

RANDOM EXCITATION

SHOCK SPECTRA
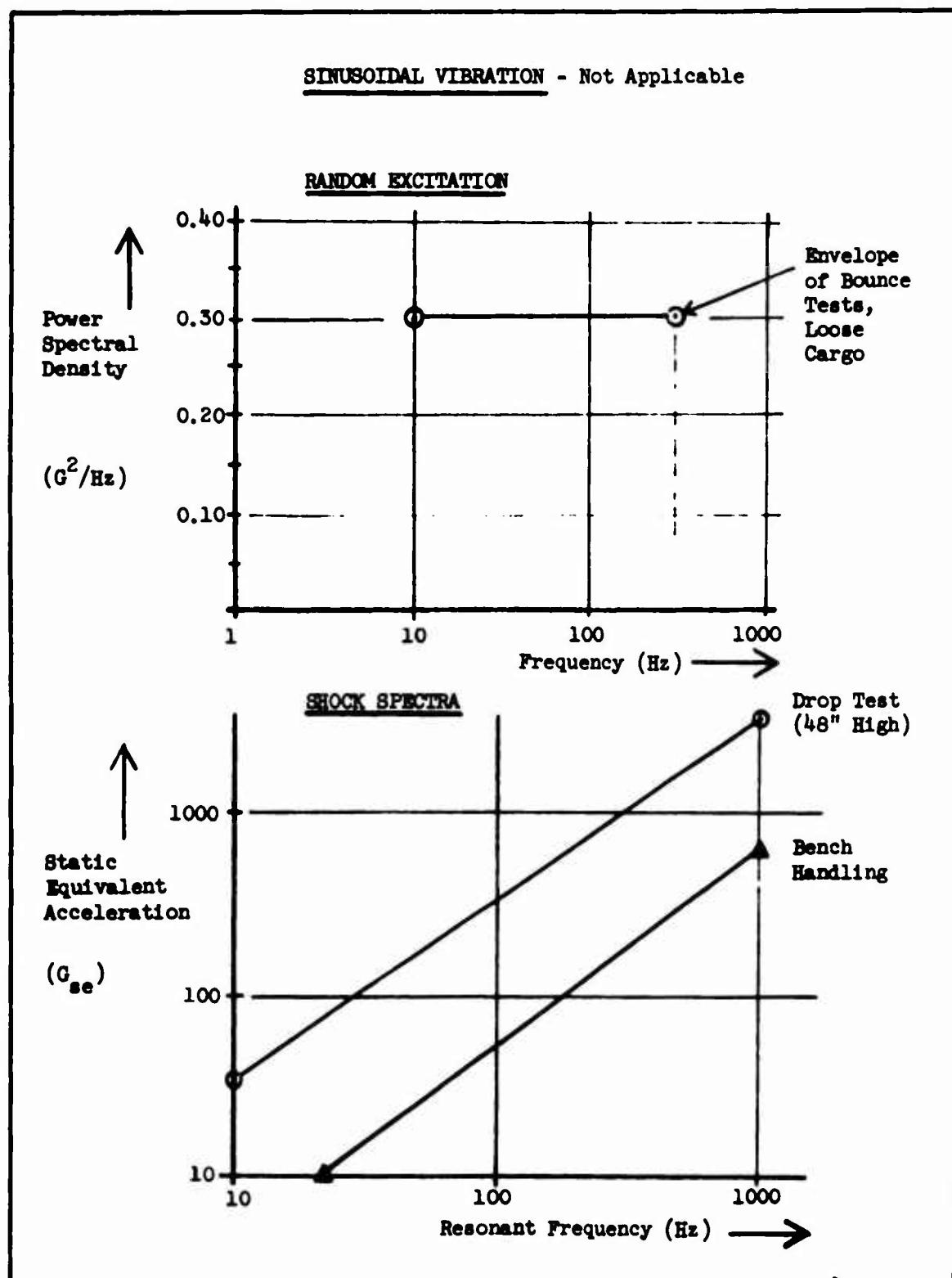
INPUT EXCITATIONS FOR EQUIPMENT CLASS II

INPUT EXCITATIONS FOR EQUIPMENT CLASS III

SINUSOIDAL VIBRATION - Not Applicable

RANDOM EXCITATION

Power Spectral Density (G^2/Hz)

Envelope of Bounce Tests

Munson (Wheeled Vehicle)

Frequency (Hz)

SHOCK SPECTRA

Static Equivalent Acceleration (G_{se})

Drop Test (48" High)

Bench Handling

Frequency (Hz)

INPUT EXCITATIONS FOR EQUIPMENT CLASS IV

SINUSOIDAL VIBRATION — Not Applicable

RANDOM EXCITATION

SHOCK SPECTRA

INPUT EXCITATIONS FOR EQUIPMENT CLASS V

SINUSOIDAL VIBRATION — Not Applicable

RANDOM EXCITATION

Power Spectral Density (G^2/Hz)

Envelope of Bounce Tests

Typical Munson (Tracked Vehicle)

Frequency (Hz)

SHOCK SPECTRA

Static Equivalent Acceleration (G_{se})

Drop Test (48" High)

Envelope of Ballistic Shock

Bench Handling

Frequency (Hz)

INPUT EXCITATIONS FOR EQUIPMENT CLASS VI

SINUSOIDAL VIBRATION

RANDOM EXCITATION – Not Applicable

SHOCK SPECTRA

VOLUME II

ANALYTICAL PROCEDURES

SECTION 3 - DYNAMIC RESPONSES

- The One-Degree-of-Freedom Idealization
- Structural Response to Sinusoidal Vibration Inputs
- Structural Response to Random Excitation
- Structural Response to Shock Inputs
- Vibration Response of Secondary Components
- Shock Response of Secondary Components
- Analytical Determination of Natural Frequency
- Analytical Determination of Structural Damping
- Dynamic Characteristics of Isolated Systems
- Experimental Determination of Dynamic Characteristics
- Dynamic Responses by Equipment Class Category

THE ONE-DEGREE-OF-FREEDOM IDEALIZATION

The dynamic responses of a one-degree-of-freedom system are computed by means of appropriate transfer functions. The significant parameters are natural frequency and maximum amplification factor.


The primary purpose of idealizing the complex system is to enable its dynamic response to be computed with relative ease. Consistent with the objectives of this Design Guide (that is, to provide information for the preliminary evaluation of a design), the one-degree-of-freedom idealization of a system is sufficient for most purposes and can be implemented fairly easily. This comes about because the behavior of a simple (i.e., single-degree-of-freedom) system is completely defined by two parameters; damping and natural frequency. These parameters, which may be determined either analytically or experimentally, are used to define the transmissibility curve for the system, as shown in the accompanying diagram.

The exact shape of the transmissibility curve is given by equation 1. [1] This may be approximated by equation 2, which is the undamped transmissibility. The major point of discrepancy is at resonance, where the undamped transmissibility becomes infinite. However, an approximate curve can be constructed by truncating the infinite curve at the maximum level of $Q = 1/(2\zeta)$ which is the maximum damped amplification for small values of damping. The resulting graph is illustrated.

The transmissibility curve now serves to define the appropriate transfer functions for the different types of dynamic excitation; sinusoidal, random, and shock. Thus the system responses to the Quality Assurance tests can be determined as long as the inputs are specified. These responses can be expressed in many different forms. In order to provide unification, the responses for all types of excitation will be put in terms of maximum equivalent static acceleration (G_{se}) as a function of system natural frequency.

The major limitation in this approach is that it obscures the responses of high-frequency components. This factor is discussed in detail later in this section.

The effects of damping appear primarily in the value of the maximum amplification factor Q, and secondarily in the shape of the transmissibility curve. More damping tends to decrease the transmissibility at frequencies below $\sqrt{2}\ f_n$ and tends to increase it at higher frequencies, as indicated. The assumption has been made here that structural damping, which arises from joint slippage, internal hysteresis, and the like, can be expressed in terms of equivalent viscous damping. While this is not true in the strict mathematical sense, it has been found to be a valid working assumption for all practical purposes.

$$TR = \frac{\sqrt{1 + \left(\frac{1}{Q} \; \frac{f}{f_n}\right)^2}}{\sqrt{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + \left[\frac{1}{Q} \; \frac{f}{f_n}\right]^2}} \qquad (1)$$

$$TR = \frac{1}{\left|1 - \left(\frac{f}{f_n}\right)^2\right|} \qquad (2)$$
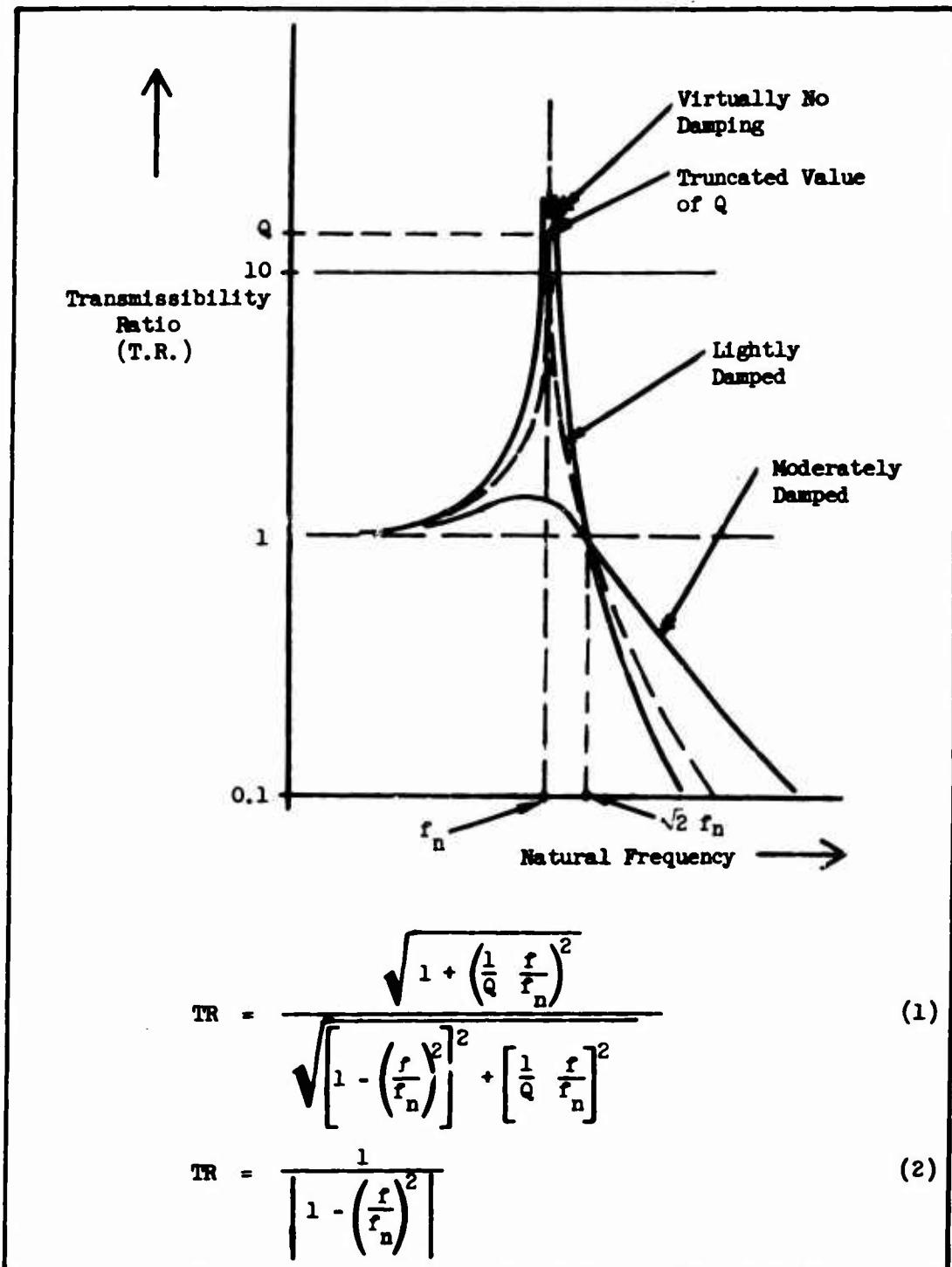
THE ONE-DEGREE-OF-FREEDOM IDEALIZATION: The basic transmissibility curve defines the transfer functions for all types of excitations.

STRUCTURAL RESPONSE TO SINUSOIDAL VIBRATION INPUTS

Sinusoidal vibration response is computed by multiplying the input by the transfer function (transmissibility).

The computation of sinusoidal response for a one-degree-of-freedom system is a relatively simple matter. Since the transmissibility curve is originally defined as the ratio of sinusoidal response to sinusoidal input, it follows that the response to any given input can be computed by multiplying the input by the transmissibility. This is illustrated in the accompanying graph. The input shown represents a typical vibration input spectrum, and is a graph of input acceleration amplitude versus excitation frequency. This graph is multiplied point-by-point by the transmissibility value at each frequency. The result is a graph showing response acceleration amplitude as a function of excitation frequency. This has been done in the accompanying sketches for two different systems, having natural frequencies f_{n1} and f_{n2}. Both systems have been assumed to have the same value of Q. It is seen that the maximum response can be computed for any given system simply by multiplying the input level at the resonant frequency by the Q for the system. Thus it is possible to determine what the maximum response of any system would be if only Q and f_n were known. If it is assumed that all typical items of USAECOM equipment have the same value of Q (say Q = 10), independent of natural frequency, then an envelope can be drawn around all possible curves of response vs frequency, as indicated. This curve, then, represents the maximum response of any system as a function of the natural frequency of the system. In that sense, each point on the curve (reproduced as a graph of G_{se} vs. f_n) represents the maximum response of a different system but with a maximum amplification factor of Q equal to 10. For systems with other values of Q, the response curve may be ratiod directly.
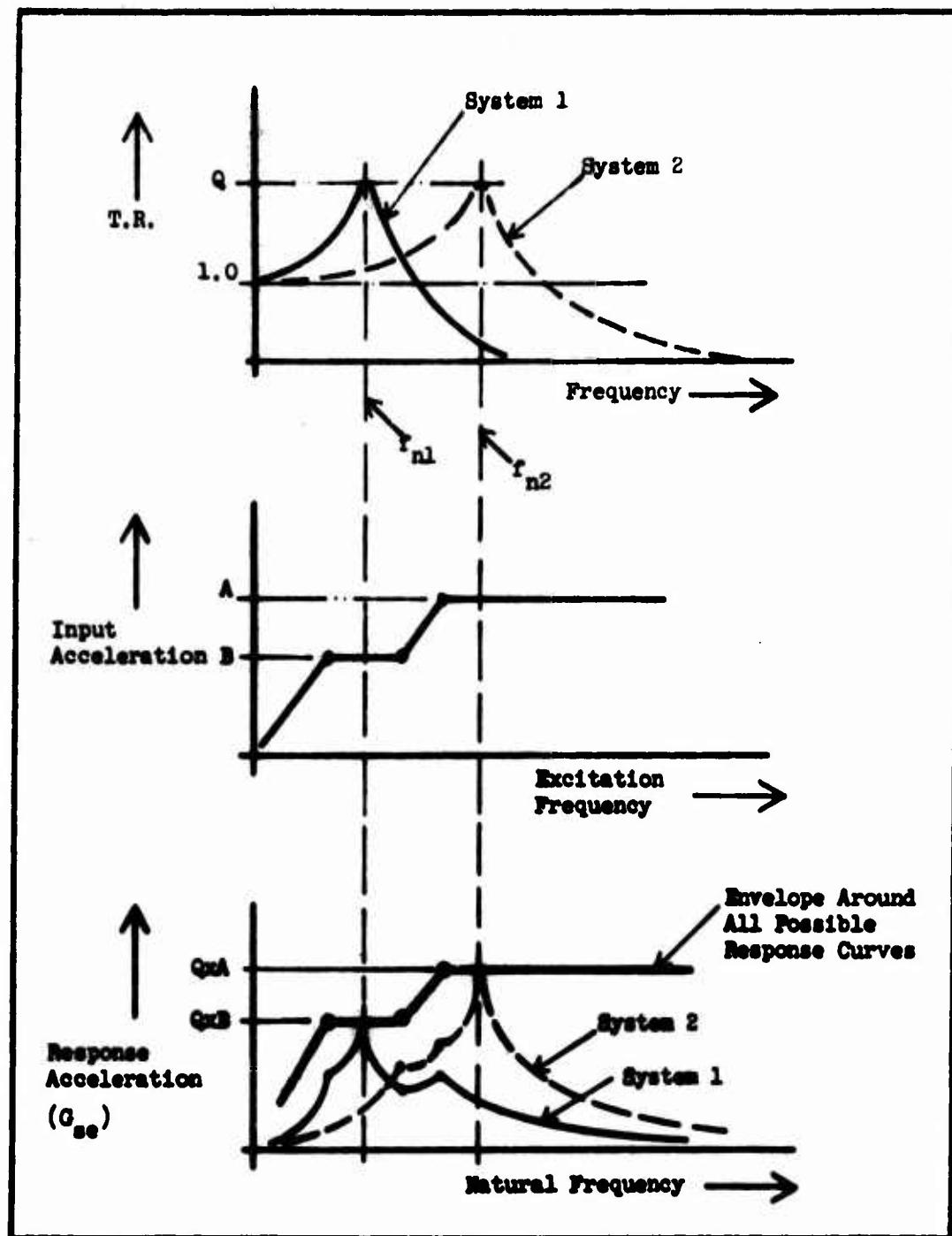
TRANSFER FUNCTION FOR SINUSOIDAL EXCITATION: Response equals input multiplied by transmissibility.

STRUCTURAL RESPONSE TO RANDOM EXCITATION

Design information can be obtained in the case of random excitation by means of appropriate computation methods involving the one-degree-of-freedom transmissibility curve.

The sinusoidal transmissibility curve is used in computing random response. However, it cannot be used simply to multiply the input, as was the case with sinusoidal excitation. This situation comes about because the random excitation cannot be described as a function of time, but rather its energy distribution is described as a function of frequency by means of a power spectral density graph. One of the properties of such a graph is that the square root of its enclosed area represents the rms level of the random signal. Thus, although the actual time history of the response cannot be determined, design information can be obtained from the power spectral density graph of the response. The necessary manipulations are performed as follows.[6] (All the steps are illustrated in the accompanying diagram.) First the transmissibility curve is squared (i.e. multiplied by itself point-by-point). Then the input spectral density is multiplied by the squared transmissibility curve. The resulting curve is the power spectral density of the response. The area under this curve is the mean-square level of the response; the rms level of the response then is the square root of the area. An approximate formula for this rms level is shown as equation 1.[6]

The accompanying diagrams show (as examples) two response curves for two different systems, both with the same Q but different natural frequencies f_{n1} and f_{n2}. The area under each curve represents the mean square value of the response acceleration for the response acceleration for the corresponding system. It is thus seen that the response characteristics of a system depend only on the Q and natural frequency of the system and the value of the input spectral density at the natural frequency. Therefore, it is possible, given Q and PSD, to determine the response as a function of natural frequency. Such a graph is shown in the form of G_{se} vs f_n. Note that each point on this graph represents the response of a different system (but all with the same Q) to the indicated input spectral density. For a Gaussian distribution of amplitudes (which is assumed here) the rms level corresponds to the one-sigma, or one standard deviation, value. This level will be exceeded 31.7 percent of the time. For design purposes, the maximum value is usually assumed to be three times as high, corresponding to the three-sigma level as shown on the graph. The probability of exceeding this three-sigma level is only 0.3 percent.

$$\text{Response rms} = \sqrt{\text{Area Under Response PSD Curve}}$$

$$\text{or rms} = \sqrt{\pi/2 \ (f_n)(Q)(PSD)} \tag{1}$$

Response of System 1:

$$G_{se1} = A_1$$

Response of System 2:

$$G_{se2} = A_2$$

Shown At Right
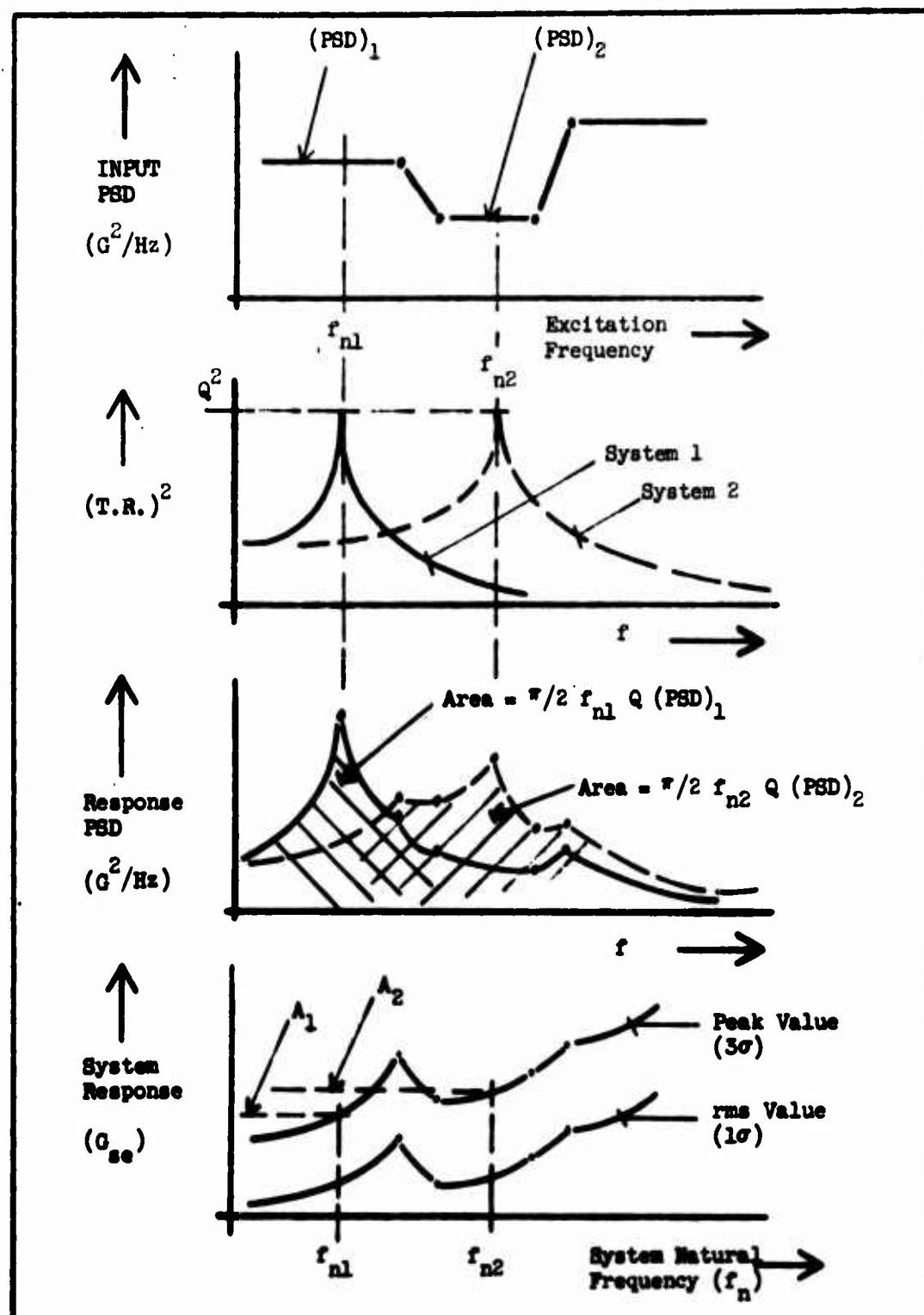
TRANSFER FUNCTION FOR RANDOM EXCITATION: Random response equals the area under a curve defined by the square of the transmissibility.

STRUCTURAL RESPONSE TO SHOCK INPUTS

Shock response is expressed in terms of maximum acceleration, which is determined on the basis of system natural frequency.

The treatment of shock phenomena is quite different from the treatment of vibratory excitation. In some instances it is possible to determine the time history of the shock input, then operate on the input mathematically to determine the time history of the response. However, it is usually easier to obtain information about shock test characteristics in the form of a shock spectrum. This is a graph of maximum response acceleration as a function of the natural frequency of the responding system. The shock spectrum of a given shock pulse can be determined mathematically by computing (for example, by using Laplace transforms) the maximum responses of many one-degree-of-freedom systems with various natural frequencies. These maximum responses are then graphed versus natural frequency, thus defining the shock spectrum. An analogous procedure can be followed physically by imposing the shock pulse on a rigid block upon which is mounted a variety of one-degree-of-freedom systems, as illustrated in the accompanying diagram. The resulting shock spectrum is shown. In a strictly correct sense, the shock sprectrum depends on system damping. However, for small values of damping the shock spectrum is reduced only slightly. This reduction will be ignored since it is minor and since ignoring it will provide conservative results.

The accompanying sketches indicate the responses of two different systems to the same shock input. It is seen that the characteristics of the responses depend on the natural frequencies. In particular, the maximum response acceleration is determined primarily by the natural frequency of the responding system. Once the shock spectrum is determined, the response of any system with the same natural frequency can be read directly.

The procedure for determining the shock response of a simple system is thus seen to involve only the determination of the system's natural frequency and the shock spectrum of the input shock.

A refinement must be mentioned with respect to shock response. The responding system will exhibit residual vibratory response in most cases. This is usually unimportant because the oscillations die out after a relatively small number of cycles. However, if the damping in the system is very small, a large number of such residual vibrations could occur. The result is that fatigue damage will accumulate and, if a large number of shock tests are performed, fatigue failure could result. This should always be kept in mind when considering shock response of a lightly damped system.

Systems

Shock Input

Response Accelera-tion

A_3
A_2
A_1

RESPONSE SPECTRUM

f_{n1} f_{n2} f_{n3}
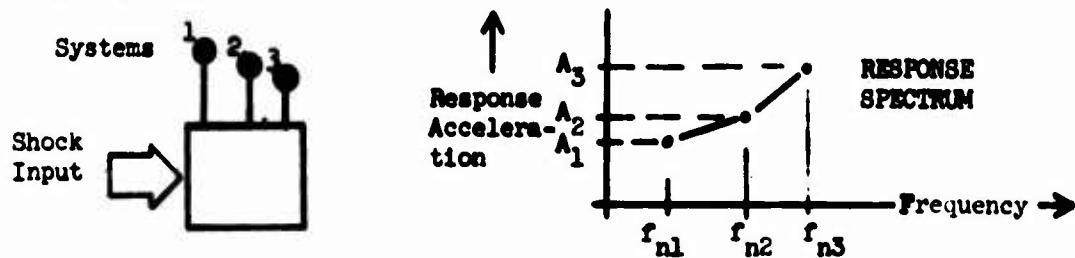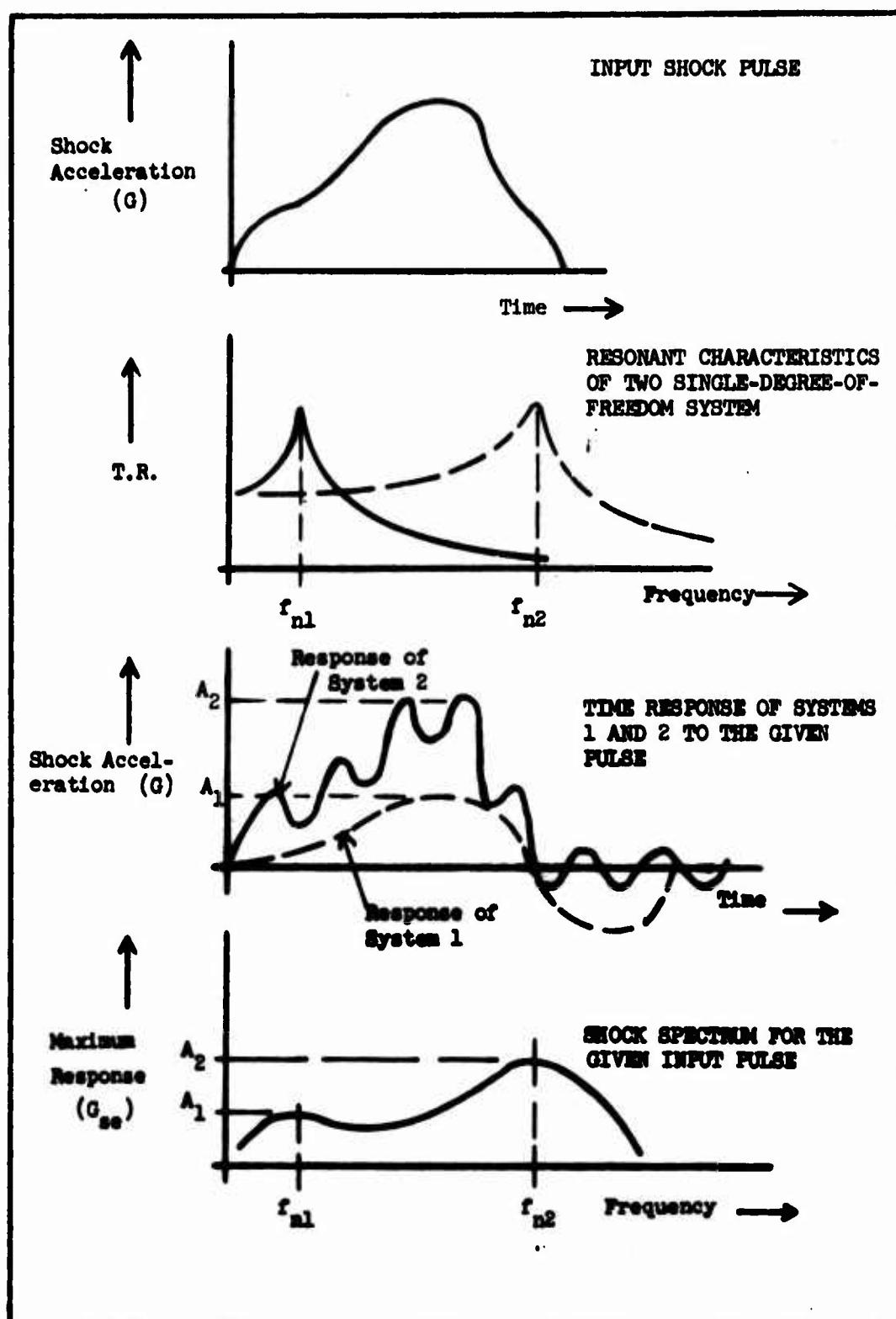
Frequency →

TRANSFER FUNCTION FOR SHOCK EXCITATION: Shock response is determined mainly by system natural frequency.

VIBRATION RESPONSE OF SECONDARY COMPONENTS

The vibration response of secondary components can be estimated on the basis of superposition of the primary and secondary transmissibility curves.

As previously pointed out, the one-degree-of-freedom idealization causes a lack of resolution with respect to behavior of secondary components. This limitation can be surmounted somewhat by idealizing a complex system as a pair of simple systems in series, as shown below. If it is assumed that the secondary system (component, system 2) is small enough that its presence does not influence the dynamic behavior of the primary system (major structure, system 1) then the behavior of the component can be described as follows. The response of the major system to a sinusoidal vibration input is given by the ususal transmissibility curve. The response of the major system is interpreted as the input into the components. This is then amplified by the transmissibility curve of the secondary system. The result is the transmissibility curve indicated, (right) which represents the response of the component to a unit sinusoidal input into the equipment.

The sinusoidal response is then computed, as previously, by multiplying the input by the compound transmissibility curve.

The random response is computed by multiplying the random input spectral density by the square of the compound transmissibility curve, and computing the area under the resulting curve. This procedure is the same as for the simple system, except that here two resonant peaks must be considered.

These calculations have been performed[2] for the general two-degree-of-freedom system. When the two critical frequencies are widely separated and the component is small compared to the major structure, the formula for the rms acceleration response of the component is given by equation 1, as noted. The assumed (3-sigma) level is then just three times the rms level, as indicated.

TWO DEGREE-OF-
FREEDOM MODEL

RESPONSE OF SYSTEM 1 ALONE TO SINUSOIDAL INPUT

RESPONSE OF SYSTEM 2 ALONE TO SINUSOIDAL INPUT

RESPONSE OF SYSTEM 2 TO A SINUSOIDAL INPUT TO THE COMPOSITE SYSTEM

RESPONSE TO A SINUSOIDAL EXCITATION

$$G_{se} = (\ddot{X}_2) = \text{Input} \times TR_{Compound}$$

RESPONSE TO A RANDOM EXCITATION

$$G_{se} = 3(\ddot{X}_2)_{rms}$$

$$(\ddot{X}_2)_{rms} = \sqrt{\pi/2 \; f_1 \; (Q_1 A_2^2) \; (PSD_1) + \pi/2 \; f_2 \; (Q_2 A_1^2) \; (PSD_2)}$$
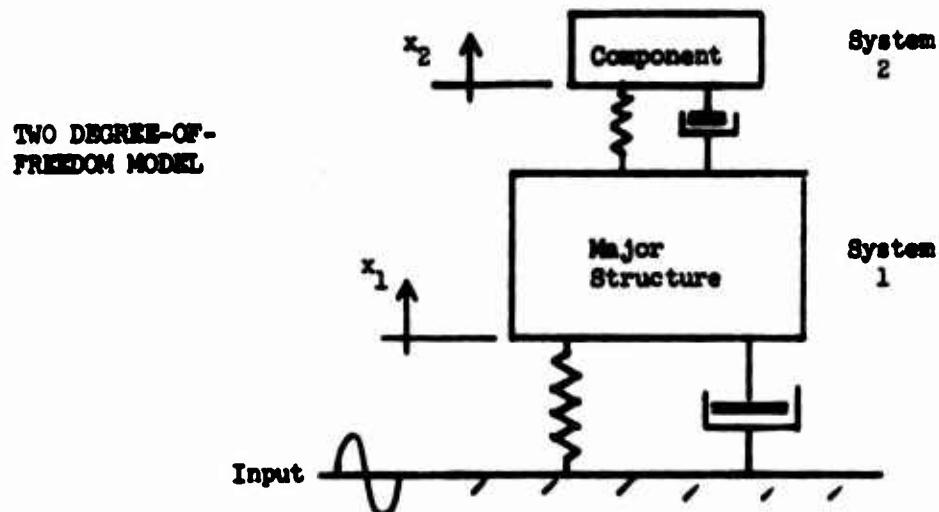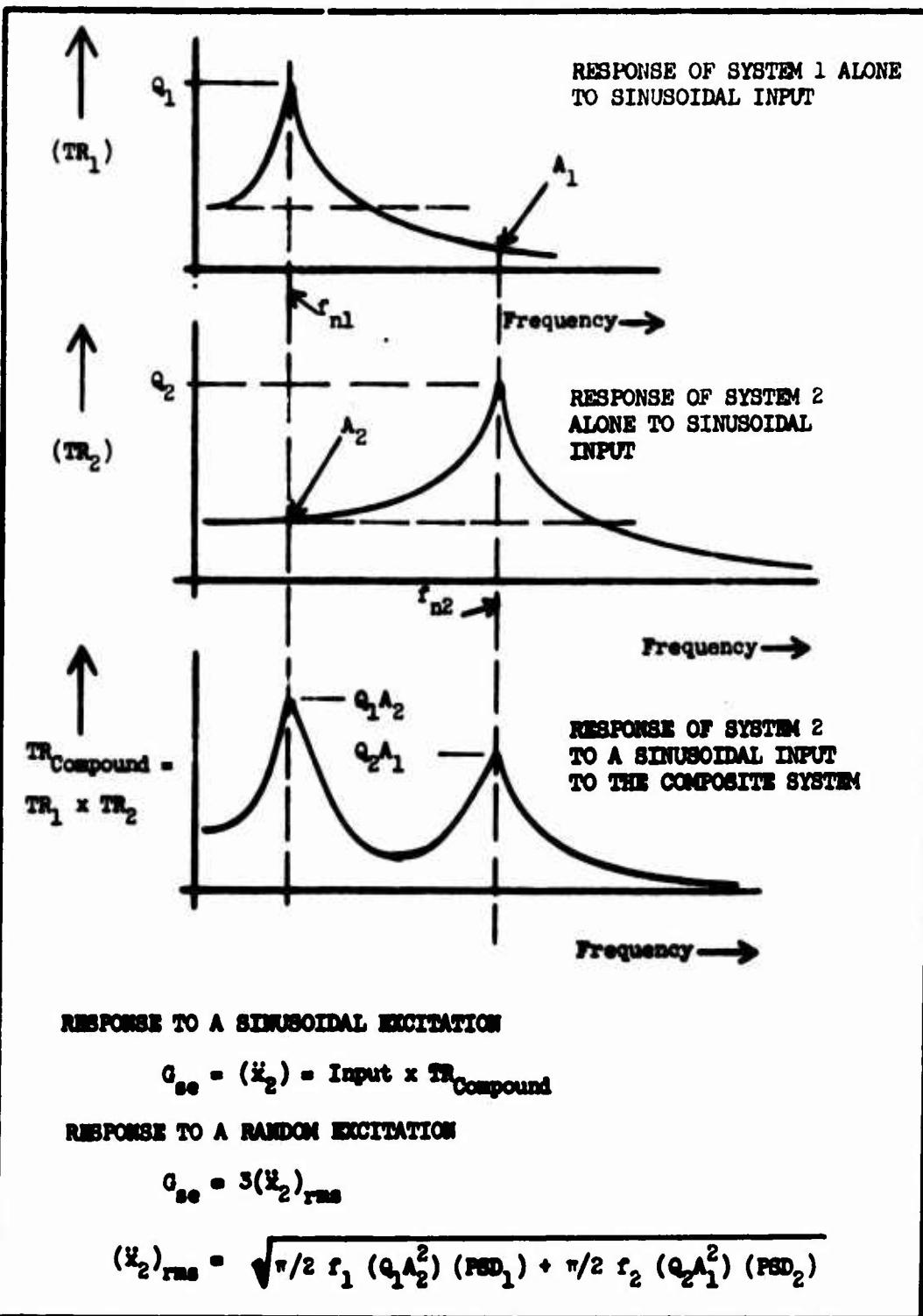
COMPOUND SYSTEM: Sinusoidal and random response of components are based on the compound transmissibility curve.

SHOCK RESPONSE OF SECONDARY COMPONENTS

The shock response of secondary components can sometimes be much more severe than is usually recognized.

A common belief is that the maximum shock amplification of a system is a factor or two. This belief is in error on two counts. It is based on the shock response of a single-degree-of-freedom system, which is a valid assumption for the analysis of major structure. However, the response of a small component depends on the degree of "tuning" between the component and the major system. The worst case occurs when the natural frequency of the component as a simple system is approximately equal to the natural frequency of the major structure as a simple system. Then two effects come into play. It is assumed that the response of the major structure is seen as an input by the component. The maximum response (also called the primary response or highest acceleration peak) of the major structure could be a factor of two times its input, if the input is a rectangular-shaped acceleration as shown.[2] The response is in the form of a sine wave, which is then amplified by a factor of 1.8 by the component.[3] The second effect occurs when the system has very little damping. Then the residual response (that is, the vibratory response occurring after the pulse is removed) of the major structure is a sinusoidal oscillation, as shown. Since its frequency is near the resonant frequency of the component, the component will amplify it as time progresses, as shown. The amplitude which finally results depends on how much damping is in the system. In general, the result is that fatigue failure could be caused by the repeated cyclic loading, whereas the usual analytical treatment is to assume only one application of a shock load.

TWO-DEGREE-OF-FREEDOM MODEL
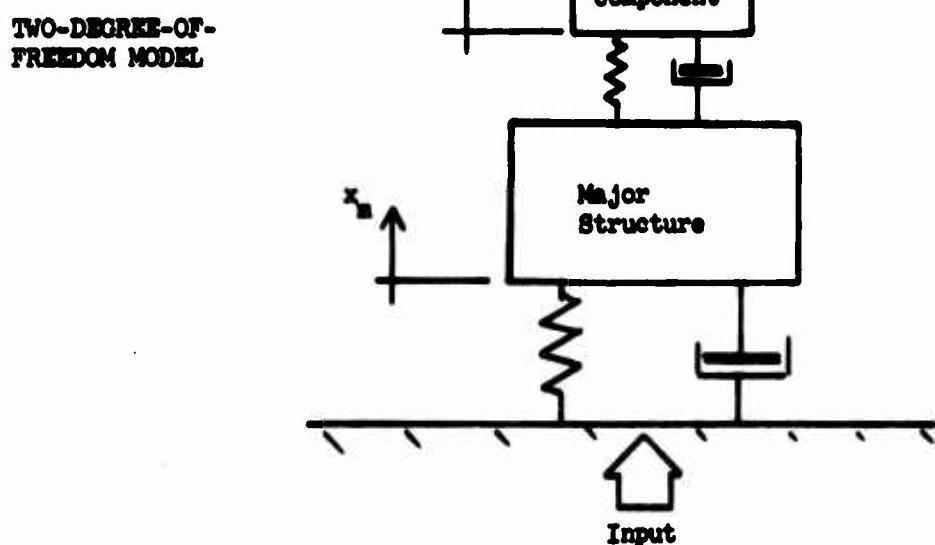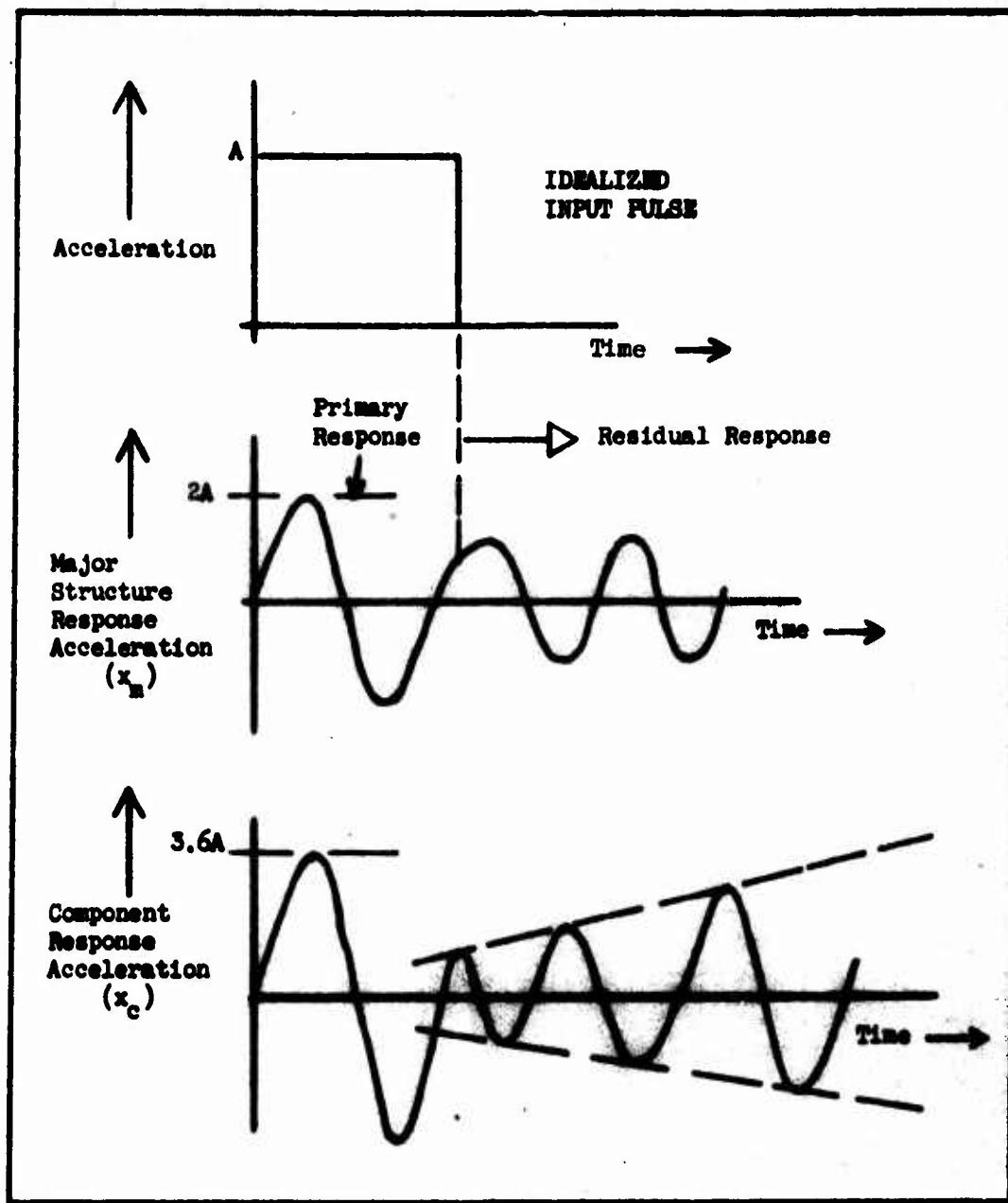
COMPOUND SYSTEM: The residual response of a component can greatly exceed the response of the major structure.

ANALYTICAL DETERMINATION OF NATURAL FREQUENCY

Several analytical methods are available for determining the natural frequency of a structural system, even before it leaves the design stages.

The natural frequency of a structure can be computed analytically. For a simple system it is given by $f_n = \frac{1}{2\pi} \sqrt{\frac{k}{M}}$. This formula can be used for a complex piece of equipment if it is transformed into slightly different form. The mass of the item is its weight divided by g: $M = W/g$. The static deflection (i.e. the deflection due to an acceleration of 1 g, or a force equal to the weight) is $\Delta = W/k$, so that the frequency formula can be rewritten as shown in equation 1 and simplified in equation 2. The force can be interpreted as acting at the unit's center of gravity or as being distributed over the unit in accordance with the actual weight distribution. The structural deformation Δ is interpreted as the displacement of the center of gravity, as shown in the accompanying diagram. Consider a uniform cantilever beam as an example. Let its total weight be W, its length and stiffness be L and EI. Under the action of a uniformly distributed total force of W lb. (corresponding to a one-g load) the deflection at the center of the beam is:

$$\Delta = .044 \; \frac{WL^3}{EI}$$

The computed natural frequency would then be:

$$f_n = \frac{3.13}{\sqrt{.044 \; \frac{WL^3}{EI}}} = \frac{3.13}{.21} \sqrt{\frac{EI}{WL^3}} = 15 \sqrt{\frac{EI}{WL^3}}$$

The exact value[1] is found to be:

$$f_n = 11 \sqrt{\frac{EI}{WL^3}}$$

A similar computation for a simply-supported beam shows the static deflection to be:

$$\Delta = .013 \; \frac{WL^3}{EI}$$

so that the computed natural frequency is:

$$f_n = \frac{3.13}{\sqrt{.013}} \sqrt{\frac{EI}{WL^3}} = 27.5 \sqrt{\frac{EI}{WL^3}}$$

compared to the exact value of:

$$f_n = 31 \sqrt{\frac{EI}{WL^3}}$$

Thus it is seen that the approximate formula is sometimes too high and sometimes too low, but usually within reasonable limits of accuracy. It is reasonable, therefore, to round off the constant 3.13 to be 3. This subject is discussed in more detail in Volume III of the Design Guide.

The foregoing determination of f_n can be used to establish the approximate transmissibility curve as indicated. The graph is drawn as a function of f according to the formula:

$$TR = \frac{f_n^2}{\left[f_n^2 - f^2\right]}$$
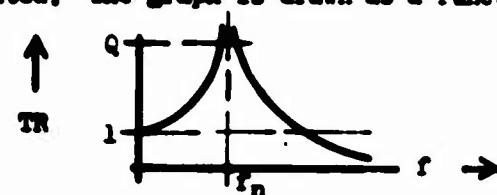
The graph, instead of being allowed to become infinitely high when $f = f_n$, is truncated at the level TR = Q. The determination of this maximum amplification is discussed in the next topic.

Specimen At Rest   Concentrated Force At C.G.   Uniformly Distributed Forces
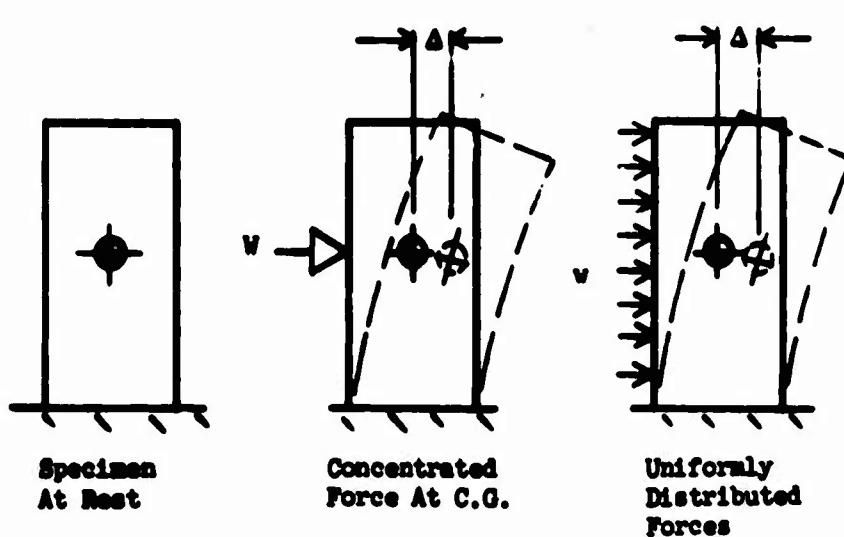
$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{kg}{W}} = \frac{1}{2\pi} \sqrt{\frac{g}{\Delta}} \qquad (1)$$

$$f_n = \frac{3.13}{\sqrt{\Delta}} \quad (f_n \text{ in Hz and } \Delta \text{ in inches}) \qquad (2)$$

NATURAL FREQUENCY: A simplified formula is used to compute an approximate value based upon deflection.

ANALYTICAL DETERMINATION OF STRUCTURAL DAMPING

Structural damping, a significant dynamic characteristic, can be estimated analytically before the design is complete.

As was seen in the foregoing sections, the dynamic behavior of a one-degree-of-freedom system is determined on the basis of its natural frequency and maximum amplification.

The maximum amplification, called Q, is determined by the amount of damping in the system. The relationship between Q and damping is $Q = 1/(2\zeta)(1)$, where ζ is the amount of damping, expressed as a fraction of critical viscous damping. This quantity really cannot be computed analytically for an actual piece of equipment, for it depends upon such factors as material hysteresis damping, energy losses in slipping joints, energy losses due to impacting parts, and damping due to air turbulence and liquid turbulence. However, ζ can be predicted on the basis of practical observations of the behavior of various pieces of equipment which have been built and tested in the past. A lower limit of $\zeta = 0.02$ applies for bolted and riveted assemblies, and a probable upper limit of $\zeta = 0.10$ is applicable to complex equipment such as a guided missile which contains moving parts. A good rule of thumb is to use a value of $\zeta = 0.05$ for a given item of equipment unless specific information is available to make a better choice. The corresponding amplification is

$$Q = \frac{1}{2 \times 0.05} = 10.$$

The appropriate value of Q can then be used to truncate the transmissibility curve, as indicated in the accompanying diagram.

Also shown is a family of transmissibility curves for various values of damping. It is seen that, although damping is beneficial near resonance, it can be harmful for excitations at frequencies higher than $\sqrt{2} \ f_n$. This should be kept in mind when considering the advisability of adding damping to a system.
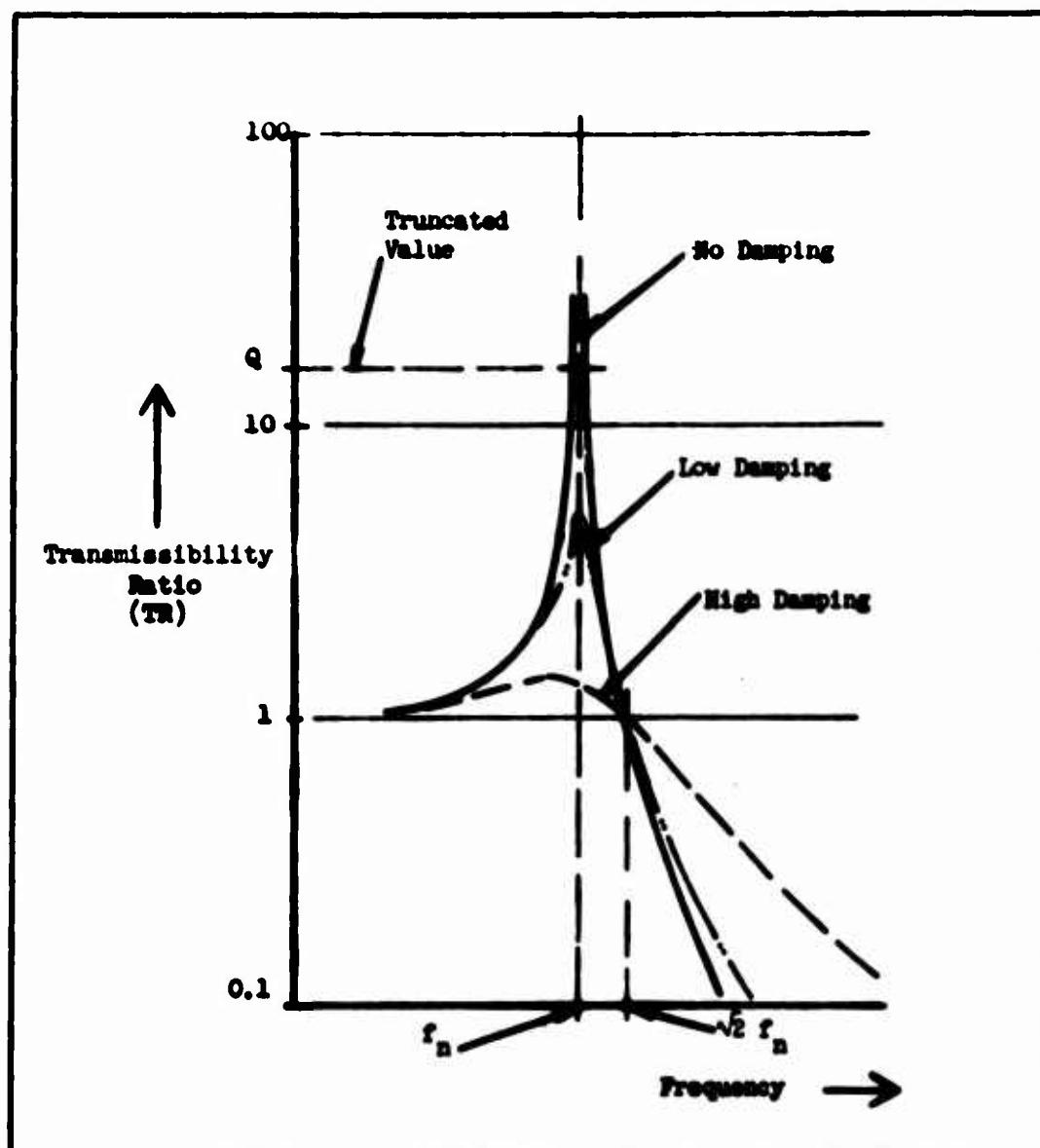
STRUCTURAL DAMPING: The maximum transmissibility depends upon how much damping is present.

DYNAMIC CHARACTERISTICS OF ISOLATED SYSTEMS

A piece of equipment, when supported on resilient shock or vibration isolators, can be treated analytically as a rigid body on discrete springs.

The equipment under discussion here generally (but not always) is in the form of a rectangular mass supported on four resilient mounts, shown in the accompanying diagram. In such a situation, two planes of symmetry are apparent: x-z and y-z. The resulting lowest modes of vibration are: 1) vertical bounce in the z direction; 2) lateral displacement (Δ) along the x-axis coupled with rotation (Θ) about the y axis; 3) lateral displacement (Δ) along the y-axis coupled with rotation (Θ) about the x-axis. The analytical determination of the fundamental resonant frequencies of this type of system is a straightforward matter. Five basic frequencies (two coupled frequencies in each lateral direction plus the vertical bounce frequency) must be computed as follows:

$$f_z = \frac{1}{2\pi} \sqrt{\frac{4k_z}{M}}$$

where k_z is the stiffness of each isolator in the z, or bounce, direction.

The chart shown in the diagram[4] provides a simplified means of determining the coupled frequencies. The information required inclues the stiffness characteristics of the springs and the radii of gyration of the item with respect to the x and y axes.

For the coupled frequencies corresponding to the translation in the y-direction and rotation about the x-axis, a value is computed for the dimensionless parameter

$$\frac{\rho}{b} \sqrt{\frac{k_x}{k_z}} \quad .$$

(Here k_x is the stiffness of each isolator in the x direction and ρ is the radius of gyration with respect to the y-axis.) Then the graph is entered, as shown, and the points corresponding to the appropriate value of a/ρ determine the values ρ/b $f/f_z(\rho)$. The two values of f are the coupled frequencies being sought. Note that f_z is the bounce frequency, determined previously.

For the coupled frequencies corresponding to translation in the y-direction and rotation about the x-axis, the procedure is similar. Only the numerical values of ρ, a, and b are changed, and k_y is used instead of k_x. As a practical matter, k_x and k_y are usually identical, and the value of "a" will not change. Thus, only ρ and b must be re-evaluated. (Note: the dimensions "a" and "b" refer to the spring spacing and position, as shown in the accompanying diagram.)
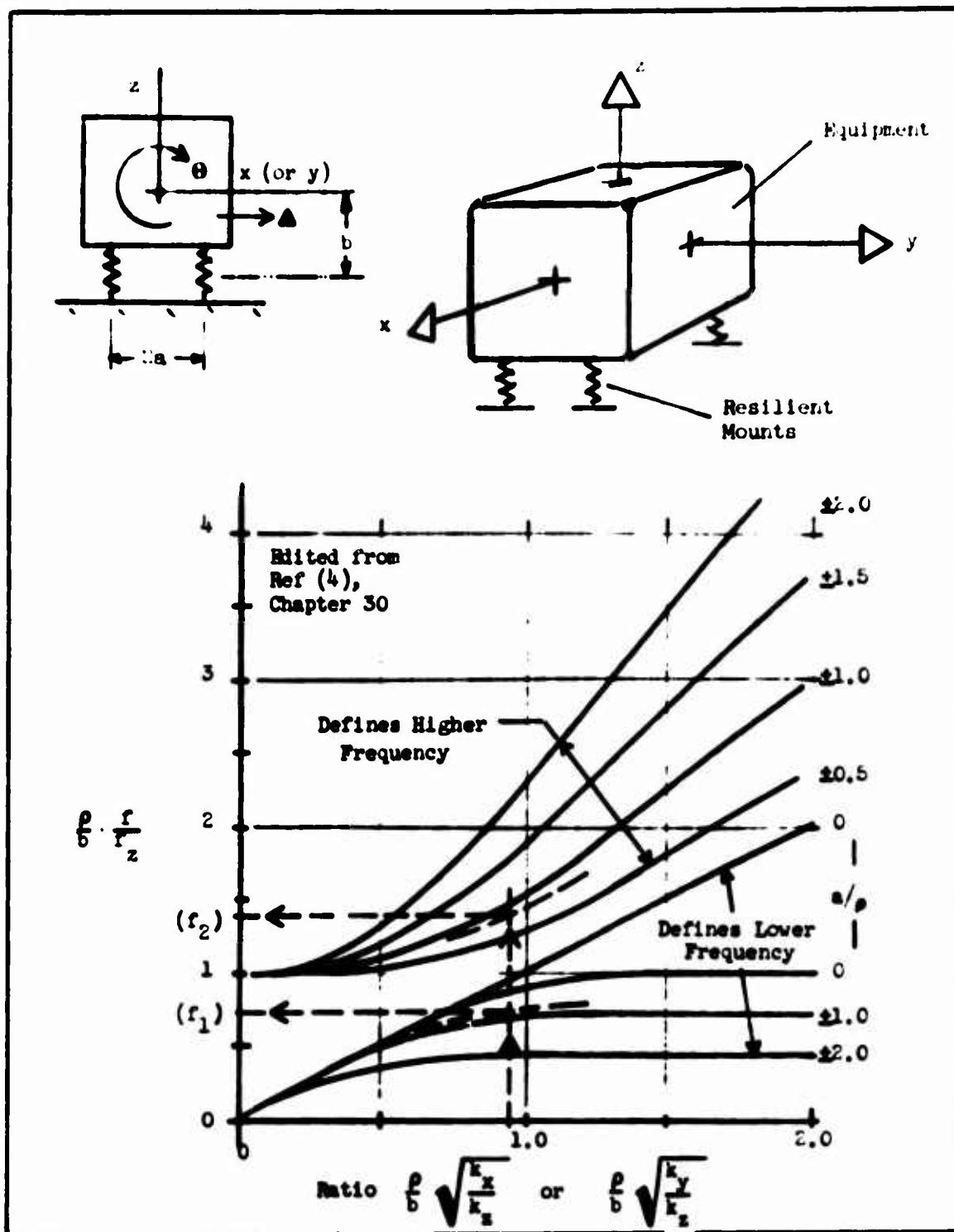
EFFECTS OF RESILIENT ISOLATORS: Several coupled natural frequencies result from the presence of isolators

EXPERIMENTAL DETERMINATION OF DYNAMIC CHARACTERISTICS

After a piece of equipment has been built, its dynamic characteristics can be determined by means of some simple measurements.

Probably the simplest means for experimentally determining the dynamic characteristics of an item of equipment is the so-called "hammer" or "twang" test. The equipment is caused to vibrate by rapping it, and the vibration level is recorded on an oscillograph as shown. After the high-frequency transients damp out, the trace becomes a damped sine wave. The frequency can be measured directly by counting the number of oscillations in some convenient time period. The damping can be computed[1] from the relationship $\zeta = \frac{1}{2\pi} \ln \frac{x_1}{x_2}$ where x_1 and x_2 are any two successive amplitudes. This is then converted into amplification according to the formula $Q = 1/(2\zeta)$. Then, with f_n and Q known, the transmissibility curve can be constructed as discussed previously.

Similar results can be obtained by means of an actual vibration test. Here the vibration response is plotted as a function of excitation frequency, so that the actual (rather than approximate) transmissibility curve is defined. The natural frequency and maximum amplification can thus be determined directly, as indicated. The instrumentation utilized to accomplish this can vary from the very simple (accelerometer output read on a meter) to the very sophisticated (impedance head between shaker and item), with output recorded on an x-y plotter.

In the event that the instrumentation required for the aforementioned test procedures is not available, similar results can be obtained by means of a stroboscopic light and a flexible bar such as a steel ruler. The bar is attached to the equipment so that its length can be conveniently varied. Then the equipment is struck, or "twanged", several times. The unsupported length of the bar is varied until its resonant frequency is the same as that of the equipment, indicated when the bar's response is maximum. Then a stroboscope is used to determine the resonant frequency of the vibrating bar. This can be done by twanging either the equipment or the bar. The advantage of using such a bar is that its amplitudes will be larger than the equipment's, thus easing the stroboscopic observations.

Without the means for measuring damping, the maximum amplification is determined by means of judgment, as discussed previously. It will range between Q = 5 and Q = 15, depending on the structural nature of the equipment.
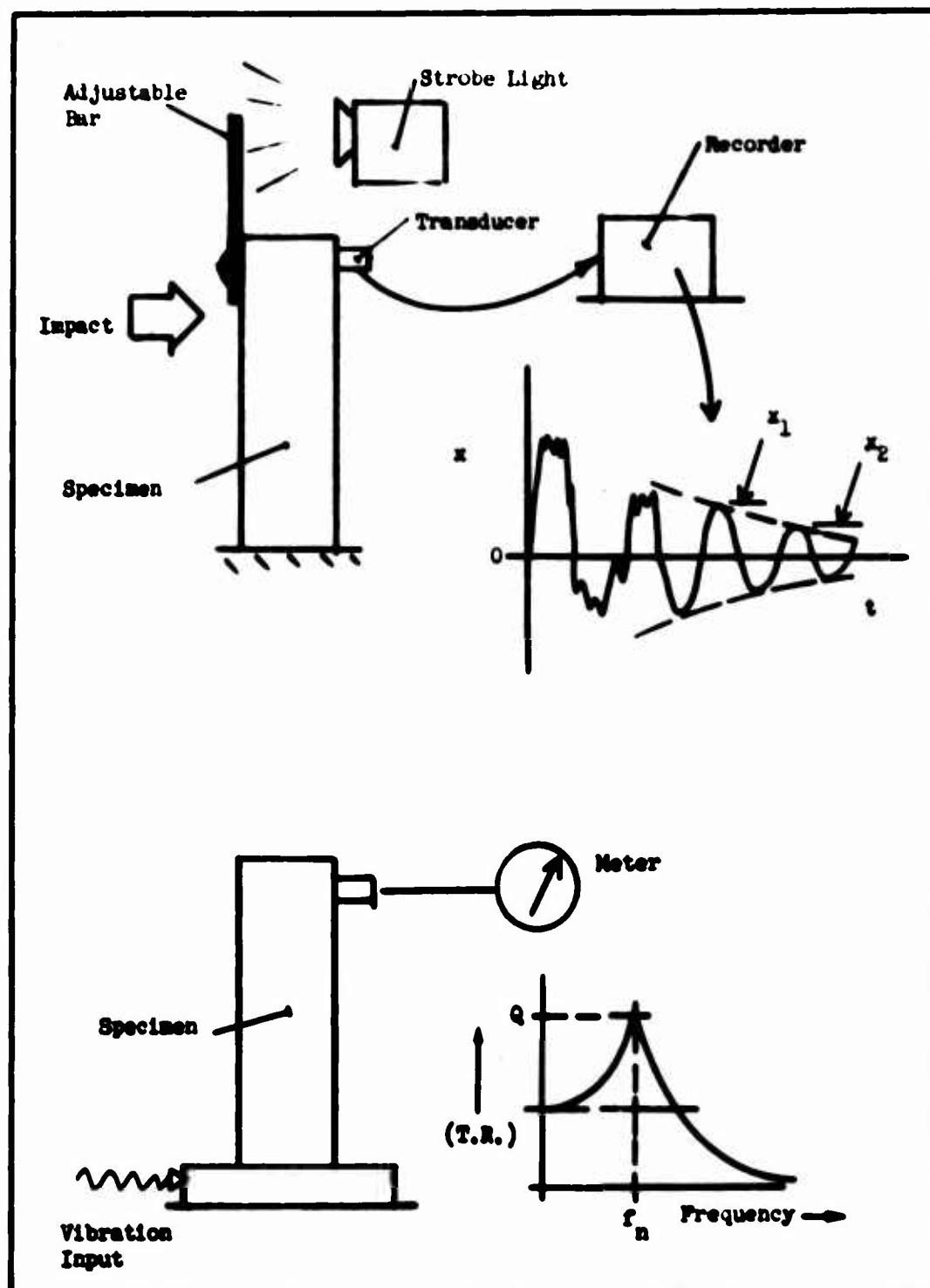
EXPERIMENTAL METHODS: Measurements of equipment dynamic behavior can indicate natural frequency and damping parameter.

DYNAMIC RESPONSES BY EQUIPMENT CLASS CATEGORY

The dynamic responses of equipment in the various equipment classes can be determined by the previously presented methods.

The dynamic inputs resulting from the USAECOM Quality Assurance tests were summarized, by class, in Section 2 of this volume. These inputs can be transformed into dynamic responses (i.e., equivalent static acceleration, G_{se}) by means of the methods presented previously in this section. The responses are presented in the accompanying graphs, which are arranged according to equipment class.

The responses to sinusoidal vibration (class VI only) were determined by multiplying the inputs by the maximum amplification factor Q. In this case, the value Q = 10 was assumed as nominal. If a particular item of equipment has a different value of Q, the results presented here may simply be scaled up or down as indicated.

The responses to the random excitations were computed by the formula

$$G_{se} = 3 \sqrt{\frac{\pi}{2} \, f_n \, Q(PSD)}.$$ The value of Q = 10 was assumed, so that

the appropriate formula is $G_{se} \cong 12 \sqrt{f_n \, (PSD)}$. If an item has a different value of Q, the results presented can be scaled up or down by the square root of the ratio of Q's.

The shock responses are read directly on the shock spectrum graphs, by definition. Thus the shock response graphs presented here are simply reproductions of the spectra presented previously. The only system parameter of significance here is natural frequency, so that the results presented are applicable to all systems regardless of the value of Q.

In summary then, it is seen that the dynamic responses have been determined according to the scheme:

$$G_{se} = \text{input x transfer function}$$

where the forms of the input and transfer function were dependent on the nature of the excitation - vibration, random, or shock.
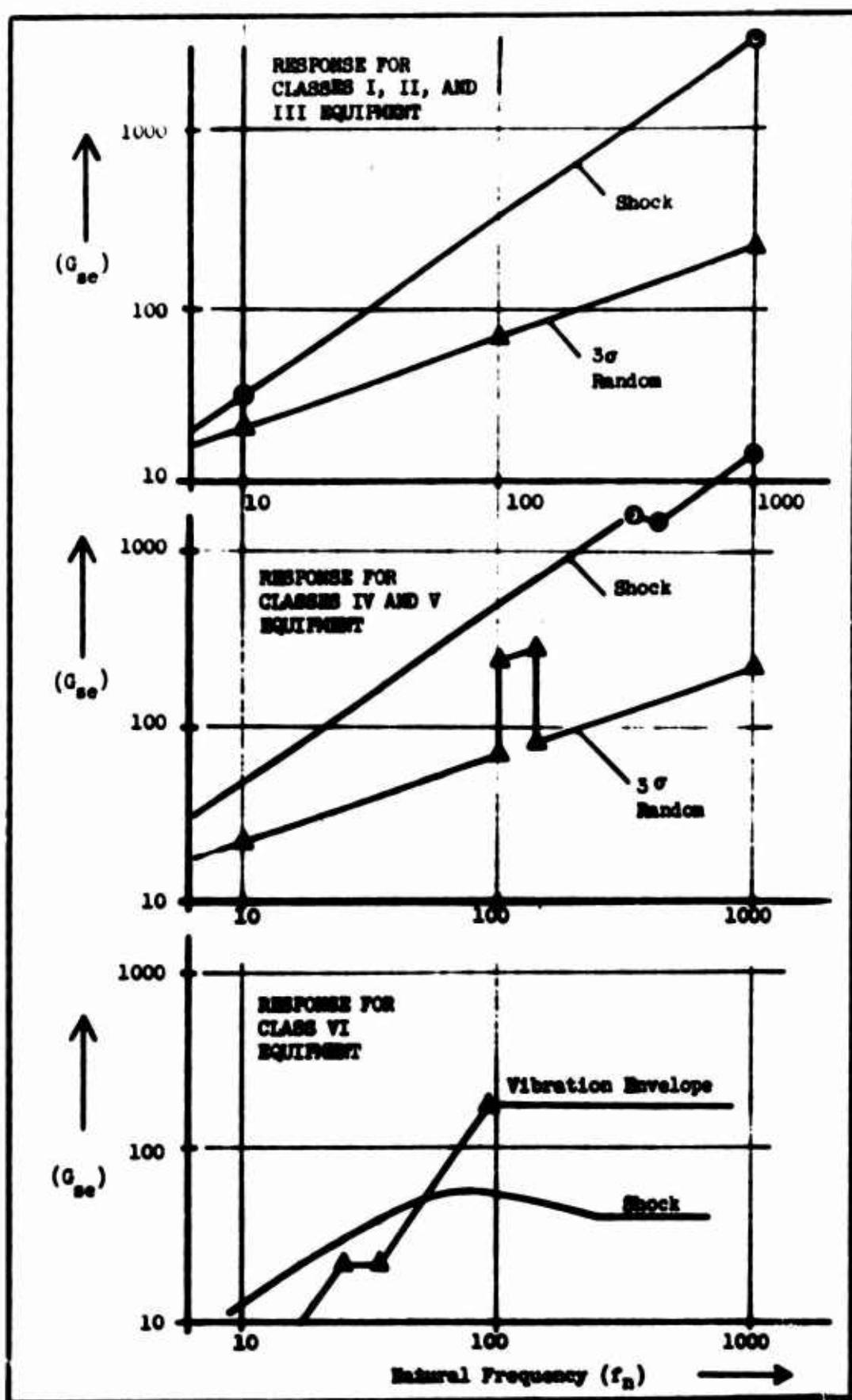
VOLUME II

ANALYTICAL PROCEDURES

SECTION 4 — STRENGTH DETERMINATION

- Factors Affecting Strength Calculation

- Simple Overload Failure

- Defining Fatigue Failure

- Designing for Infinite Fatigue Life

- Using the Concept of Fragility to Define Equipment
  Strength

FACTORS AFFECTING STRENGTH CALCULATION

In order to evaluate a design with respect to structural adequacy, it is first necessary to determine its strength. This determination depends on the mode of failure and is expressed in terms of fragility.

The ultimate objective of this volume is to provide the tools for evaluating the structural adequacy of a particular design. This evaluation is based on the strength of the design, where strength is defined as the quantitative measure of the ability of the structure to perform its function in the presence of adverse mechanical loading. Such strength determination involves the relationship between loading and material characteristics, and considers the various possible modes of failure anticipated.

Material properties and type of loading generally determine the maximum load which may be applied to the structure. Many different modes of failure must be considered, each mode being associated with a specific type of loading and a particular material property. For example, buckling depends upon the maximum value of external load and the modulus of elasticity of the material, as well as the structural configuration. On the other hand, fatigue failure is governed by the number of times the loading is applied, the magnitude of the loading, and the fatigue strength of the material. Other modes of failure must be considered also, of course. To bring order into the potentially chaotic list of failure modes, it is convenient to group them according to the type of loading to be imposed. Thus only two categories are required: maximum one-time loads and loads which are applied in a repetitive manner. Those failures associated with the former include yield, rupture, buckling, and excessive deformation. The latter produce only one mode of failure: fatigue.

As if to complicate the situation further, it must be pointed out that the loading magnitude is not usually defined specifically, but rather is a function of the structure's natural frequency. Thus the overload strength of the structure varies with natural frequency also. Going one step further, consider the case of fatigue strength. If an oscillating load is applied for a specified time, then more cycles of stress result if the oscillation frequency is high than if it is low. Thus the fatigue damage will be greater for the higher frequency system. This dependence on frequency gives rise to the concept of fragility, which is simply a means of displaying graphically the variation of strength of a design with natural frequency.
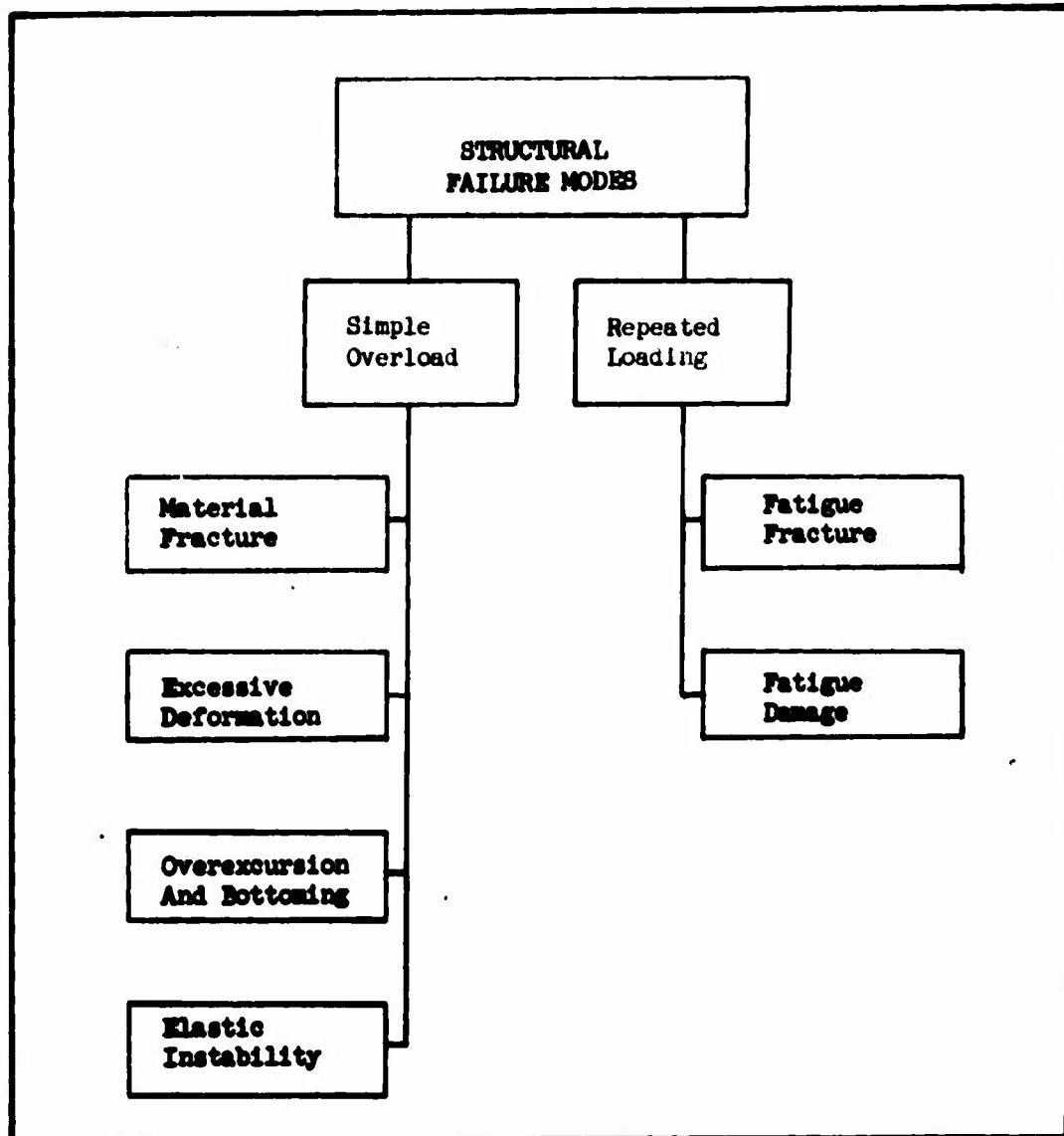
STRUCTURAL FAILURE MODES: There are many facets to the definition of a realistic failure mode resulting from shock and vibration. All failure modes may however, be grouped into one of two categories.

SIMPLE OVERLOAD FAILURE

Certain types of failure result from a single application of an excessively high load. These include fracture, excessive deformation, and instability.

The mode of failure identified as "simple overload failure" corresponds to failure caused by just one load application, as opposed to the fatigue phenomenon. Several types of failure can result from a single application of an excessively high load. These include yield, rupture, instability, excessive excursion, and isolator bottoming. The particular failure which actually occurs in a given situation depends upon the structural configuration and the mechanical properties of the material involved.

The starting point in determining the anticipated mode of failure is to describe the anticipated dynamic loadings. This should be done not quantitatively but qualitatively, to determine the points of application of inertia loads and their directions. The inertia loads of interest (those associated with this type of failure) are identified as "static equivalent acceleration", or G_{se}, as presented in Section 3 - Dynamic Responses.

It is assumed that the structure under study has been defined well enough to allow the determination of stress levels in the various structural elements resulting from the application of specific forces. This being the case, detailed stress analysis methods are invoked to determine the stresses and displacements due to loads applied to the structure in a manner similar to the anticipated inertia loads. The results are then classified according to three parameters: displacements, tensile (and shear) stresses, compressive (and shear) stresses.

The computed displacements will be compared with the established values of allowable displacements to determine the margin of safety. Two types of displacements must be considered in discussing modes of failure. The first, caused by structural deformation, leads to mechanical interferences between neighboring components within an item of equipment or between neighboring items themselves. The second arises when an item of equipment is mounted on resilient isolators to protect it from large accelerations during shock and vibration. In such a case, the item will usually move like a rigid body with deflections caused by deformation of the isolators. Not only could this cause interference between neighboring items, but of even more seriousness, an isolator can reach the limit of its resilient travel and "bottom out". This causes large dynamic forces on the equipment, thus negating the beneficial effects of the isolators.

The modes of failure caused by excessive tensile (and shear) stresses are called "yield" and "rupture". These phenomena depend only upon the strength characteristics of the structural materials. The allowable stresses are independent of the configuration of the structure. Yielding is considered to have occurred when the stress exceeds the yield strength of the material. Rupture constitutes actual separation of the material, and will occur when the stress reaches the tensile (or shear) strength of the material. It should be pointed out that, while rupture per se constitutes failure, such may not be the case with yielding. Rather, in some instances, yielding might be allowed if the displacements are not

excessive. Thus the specific requirements of each particular design will determine whether yielding is to be considered a mode of failure.

The modes of failure caused by excessive compressive (and shear) stresses are "yield", as discussed previously, and "buckling". Buckling, also known as instability, occurs in the form of structural collapse (without rupture) when the applied loads exceed the level at which the structure becomes unstable. The most common form of buckling is the one which occurs in an axially loaded column. Less frequently seen is the buckling of the thin skin of a shell-like structure. The buckling mode of failure differs from the others in the respect that the critical stress is not dependent upon material properties alone. Rather, the structural configuration itself is the major governing factor in determining the critical stress. The only material property of interest here is the modulus of elasticity. Some typical formulas for critical compressive stress are:[5]

Pin-ended column: $\quad S_{cr} = \dfrac{\pi^2 E}{(1/r)^2}$

Axially loaded cylindrical shell: $\quad S_{cr} = 0.3 \dfrac{Et}{R}$

Other instabilities involve such buckling modes as lateral deflection of beams and torsional deflection of columns. Chapter 1, Volume III of this Design Guide presents a complete treatment of the buckling phenomenon and lists the formulas for critical stresses for many structures.

DEFINING FATIGUE FAILURE

Repeated loading can cause failure at stress levels significantly lower than the material yield strength.

Situations frequently arise, as have been observed repeatedly, in which structural elements fracture unexpectedly when the applied stress level is relatively low. Such failures have historically occurred after considerable length of service, thereby giving rise to the label of "fatigue" failure. It was first thought, because such fractures had a crystalline appearance with no evidence of plastic deformation, that "crystallization" had caused a reduction in strength. Subsequent research indicated that the cause was cyclic stressing, and proved that repeated loading can cause failure at stress levels significantly lower than the material yield strength. Thus fatigue failure is defined as the sudden rupture of a structural element due to cyclic loading at stress levels below those required to cause static failure.

The simplest physical model for discussion of fatigue failure is repeated tensile loading at constant amplitude. Most available test data have been generated with this loading scheme. The tests are performed by measuring the number of cycles required to cause failure at each of several stress levels. The data points are plotted on a graph of stress versus cycles, as shown.[5] The resulting curve is a failure line such that combinations of stress and number of cycles above the curve indicate failure, and vice versa. This curve thus establishes the allowable cyclic stresses for each particular material. When plotted on semi-log graph paper (stress versus log cycles) the curves exhibit characteristics which give rise to certain convenient definitions. Most steels exhibit an asymptotic stress level below which the material can be stressed indefinitely without failure. This stress is known as the endurance limit, S_e. Most aluminums, on the other hand, do not exhibit such an asymptote: the stress decreases indefinitely (within practical ranges of cycles) as cycles increase. Thus the endurance limit quoted for aluminums is usually the fatigue strength (allowable stress) at 500 million cycles.

Many practical situations arise in which a structural element is subjected to a complex pattern of stress reversals. The complexity of significance is one in which the stress level varies from cycle to cycle. Two such circumstances are frequently encountered in service; swept sinusoidal vibration and random vibration. Several mehtods are available for determining the number of cycles at each predetermined stress level in both cases. Armed with this information, another concept must be introduced to make it useful: cumulative damage. Let the life at stress S_1 be N_1 cycles. If a structure is subjected to n_1 cycles at stress S_1 then the remaining number of cycles allowed at that stress is $(N_1 - n_1)$. Thus, a fraction of its life corresponding to n_1/N_1 has been used up. If, after the N_1 cycles at stress S_1, the structure is subjected to n_2 cycles at stress S_2, then the additional fraction of its life used up will be n_2/N_2 where N_2 is the life at stress S_2. If the sum of the life fractions (also called damage fractions) is greater than unity, then failure is anticipated. Mathematically, this is stated as follows: $n_1/N_1 + n_2/N_2 > 1$ means failure.
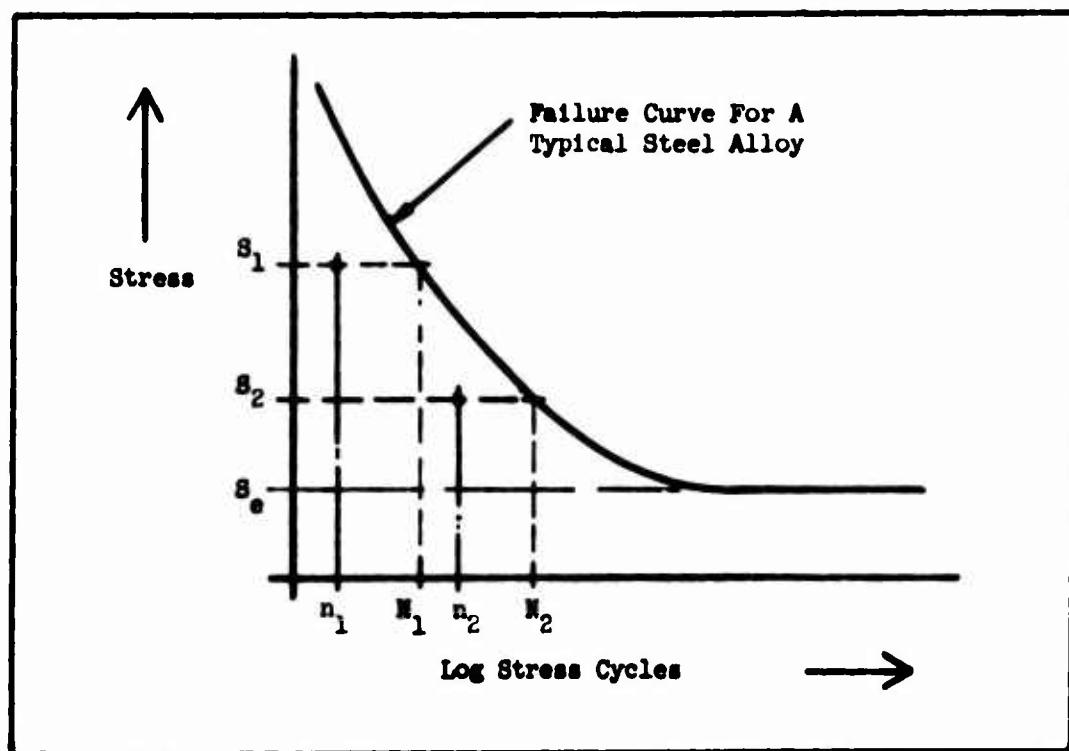
THE FATIGUE FAILURE CURVE: Low stress, repeated a sufficient number of times, can cause failure.

DESIGNING FOR INFINITE FATIGUE LIFE

Providing for infinite fatigue life offers a significant simplification for the process of design evaluation.

Because of all the complexity associated with fatigue analysis, it is recommended that a simplification be invoked in order to assure consistency with the objective of this Design Guide: to provide a simple set of tools for use by project engineers. The simplification referred to involves the subject of finite fatigue life. Design evaluation for finite life requires exact knowledge of both stress history and material properties (i.e., S-N curve) and demands complicated analysis methods. Evaluation for infinite life requires only that the material endurance limit be known and does not involve the concept of cumulative damage or knowledge of how many cycles of stress are to be imposed (an infinite number of cycles is assumed). Thus the project engineer, in keeping with his role as an evaluator rather than as a detail designer, should evaluate the fatigue adequacy of a design on the basis of infinite life — that is, by comparing the maximum oscillatory stress to the endurance limit of the material without regard for the number of cycles actually anticipated.

The result of this type of approach is always conservative, in the sense that margins of safety are computed to be smaller than they actually are. However, it should be noted that the design of USAECOM equipment is generally governed by shock loading (see Section 3 - Responses), especially those items with high natural frequencies. For most aluminum alloys[9] the ratio of fatigue strength (at 500 million cycles) to ultimate tensile strength is approximately 0.25, so that the 3σ vibration level for infinite life governs the design whenever it exceeds 0.25 times the shock response level. This situation is illustrated in the accompanying diagram. When the design is dictated by fatigue, there are two choices within the scope of this Design Guide. First, the design can be evaluated for infinite life, the penalty being excessive conservatism. Second, expert assistance can be sought from an analyst skilled in the methods of finite-life fatigue analysis.

Although this is not intended to be a complete treatise on fatigue, it should be pointed out that many other factors must be considered in a detailed fatigue analysis. Some of these are: stress raisers, heat treatment, notch sensitivity, temperature, structure size, surface finish, corrosive environment, residual stress, grain direction.[7]

The previous discussion was based on a stress history consisting of complete stress reversals, mainly because this is the type of loading actually anticipated. In general, such is not the case. Rather, the alternating stresses are usually superimposed on some mean stress. The subject is presented at this point for the sake of completeness and to indicate that methods are available to treat this case also.[5,7]
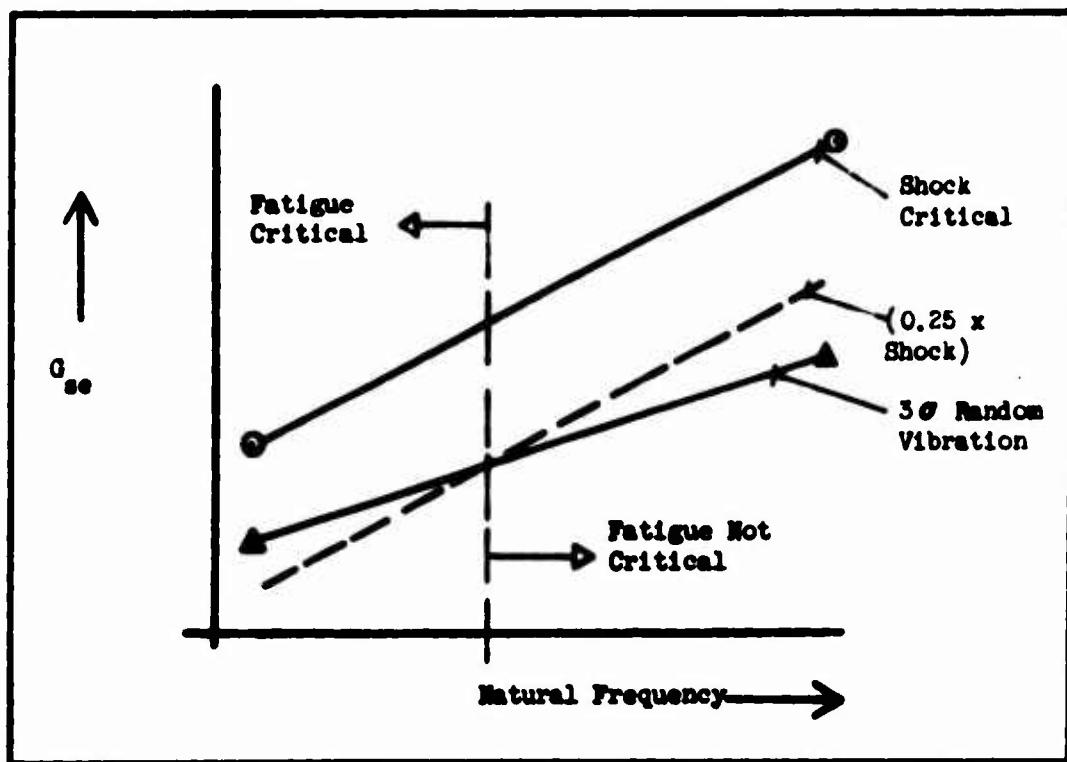
DESIGN OBJECTIVE FOR FATIGUE: Stress levels lower than the endurance limit will allow infinite life.

USING THE CONCEPT OF FRAGILITY TO DEFINE EQUIPMENT STRENGTH

The fragility of an item defines the allowable loadings which can be imposed on it. The fragility will be presented in this document as a graph of allowable static equivalent acceleration versus natural frequency of the item.

The structural adequacy of an item of equipment is evaluated by comparing its strength with the loadings imposed on it. Such a comparison is seen to be very complicated because many different failure modes are involved in the strength, and the loadings are not discrete but rather vary as a function of the natural frequency of the item. Thus, in order to simplify the evaluation process, it has been found convenient to define both strength and loading as functions of natural frequency. This makes it possible to exhibit both strength and load graphically on the same chart and to compare them at each natural frequency to determine structural adequacy as a function of natural frequency. The strength graph, expressed in terms of allowable dynamic response (G_{se}) versus item natural frequency (f_n), is known as the fragility. It should be recognized that there will be a different fragility curve for each different mode of failure. This aspect will be treated in the discussion of how fragility is computed.

In order to prevent confusion, it must be pointed out that the aforementioned definition is by no means universally agreed upon. The fragility concept was originally introduced in conjunction with protective packaging requirements. There the fragility was defined in terms of dynamic inputs and excitation frequency, for those are the significant quantities for that application. For evaluation of structural adequacy, however, the definition presented in this Design Guide may be more suitable.

The full usefulness of the fragility concept can only be realized when it is determined and used quantitatively. The quantitative measure of the fragility of an item is its maximum allowable dynamic response, expressed in terms of static equivalent acceleration, G_{se}. This allowable load is determined by means of detailed stress analysis, and is based upon the strength characteristics of the item. The force required to cause failure is calculated and is then transformed into G_{se} by dividing by the weight of the item. This is plotted graphically as a function of system natural frequency so that it can be compared with the responses computed in Section 3.

Each different mode of failure has associated with it a different fragility level. Simple overload failure (except for excessive deformation) depends upon maximum applied force, independent of natural frequency, so that its fragility is equal to G_{se}, as shown in the accompanying diagram.

The fragility curve for excessive deformation varies with frequency because the parameter of interest is displacement rather than acceleration. Thus, since vibratory displacement and acceleration are related as:

displacement x $(2\pi f)^2$ = acceleration; it is seen that a given allowable displacement corresponds to a low acceleration at low frequency and a high acceleration at high frequency. This is illustrated on the accompanying diagram.

The fatigue phenomenon depends upon both acceleration level and number of
cycles. However, since the design philosophy outlined here is that
fatigue will be evaluated for an infinite life, the allowable stress level
is the material's endurance limit. Thus the fragility is independent of
number of cycles and frequency, as shown.

One additional mode of failure should be considered: functional failure of
some small component. In this respect, the response of the structure
represents input into the component. Thus, the allowable level depends
upon the resonant frequency of the component: only small levels can be
tolerated at the resonant frequency of the component. This situation is
also illustrated.

It is seen that the fragility curves constitute a set of graphs of G_{ae}
versus f_n. An envelope curve can be constructed, as shown, to display
the absolute fragility of the item, independent of failure mode. This
envelope defines the maximum allowable dynamic response of the item as a
function of natural frequency. This can be compared with the actual
responses induced by the Quality Assurance tests to determine the struc-
tural adequacy of the design.
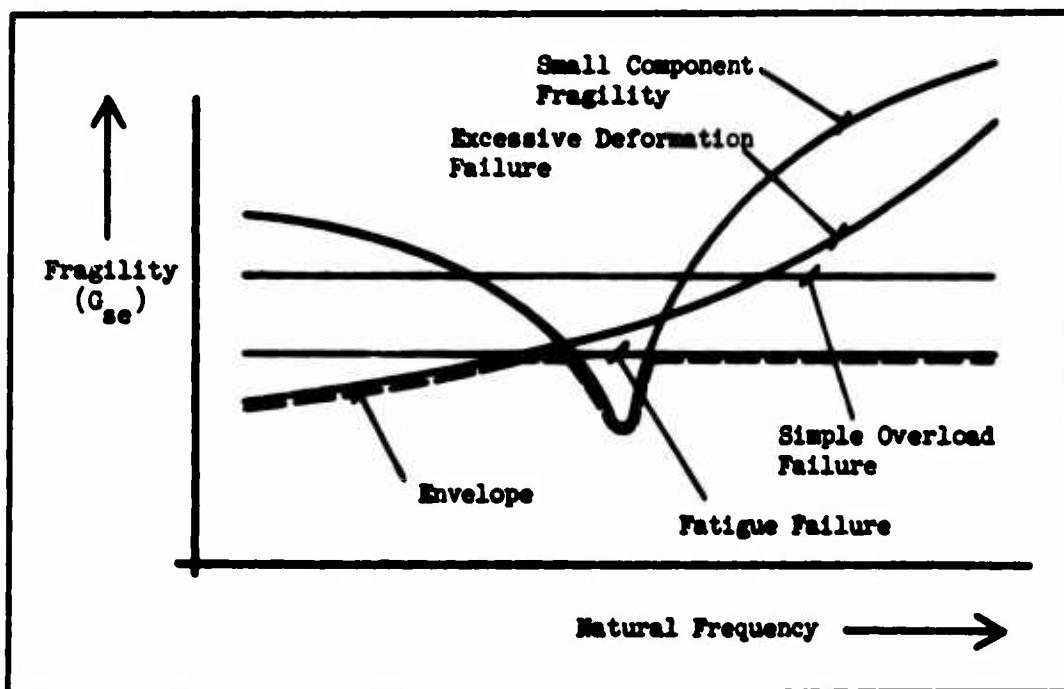
FRAGILITY CURVES: An enveloping curve may be used to define an absolute
fragility.

VOLUME II

ANALYTICAL PROCEDURES

SECTION 5 - DESIGN EVALUATION

- Fragility and Loading Curves

- Margin of Safety

- Determination of Corrective Actions

FRAGILITY AND LOADING CURVES

In order to evaluate a design with respect to structural adequacy, its strength
must be compared with its anticipated loadings.

The evaluation of a design involves many factors concerning its perfor-
mance, appearance, and other attributes. With respect to structural
adequacy, however, the only attribute of concern is the ability of the
item to resist the applied loads without failure or excessive deformation.
Evaluation of the structural adequacy involves comparison of the strength
with the anticipated loadings. This comparison process can be performed
either qualitatively or quantitatively, and usually both approaches are
taken. The qualitative comparison consists of simply examining the
item's fragility and loading curves. The quantitative comparison is
accomplished in similar fashion at the beginning, but it is extended to
the point of computing a numerical margin of safety. This makes it
possible to determine just how adequate (or inadequate) an item may be.
Such information is required to decide whether an inadequacy can be
corrected simply or whether major redesign is required.

The qualitative evaluation of a design is accomplished as follows:
as shown is Section 3, each class of equipment has associated with it
specific dynamic inputs and specific dynamic responses. The corresponding
graphs of static equivalent acceleration, when plotted against system
resonant frequency, constitute the dynamic response curves, as illustrated
in the accompanying diagram. A separate curve is shown for sinusoidal,
random, and shock excitation. Above these curves is drawn an enveloping
curve, as shown, which now represents the maximum dynamic responses,
independent of source.

The next step is to construct the fragility curves which, as discussed in
Section 4, represent the maximum allowable dynamic responses of the equip-
ment. Again a family of curves is encountered, one for each mode of
failure. An envelope around these curves serves to define the fragility,
or allowable dynamic response, independent of mode of failure.

The final step is to plot, on the same graph, the two enveloping curves
for dynamic response and fragility. Qualitative comparison of the two
curves shows whether or not the equipment is structurally adequate.
Adequacy is indicated when the entire fragility curve lies above the
entire loading curve, as shown. Conversely, a deficiency is indicated
whenever the loading curve crosses over and exceeds the fragility curve.
Deficient regions are identified as shown in the accompanying diagram.

It is thus seen that the foregoing procedures will lead to qualitative
decisions concerning the structural adequacy of the equipment being
evaluated. Furthermore, the information is not only of the "go, no-go"
variety, but rather the dangerous frequency ranges are also indicated.
Thus it is possible also to observe how much benefit would result from
certain design changes, such as changing resonant frequency.

The approach taken to evaluate a design on the basis of fragility and
loading curves demonstrates an additional advantage. Not only can an in-
adequate design be corrected, but a marginal design can be improved even
though it may be adequate. The consequent improvement in reliability is
one of the direct benefits of the approach presented here.

SUMMARY OF VARIOUS LOAD SOURCES

Dynamic Response (G_{se})

Load Envelope

Frequency →

SUMMARY OF FAILURE MODES

Fragility

Fragility Envelope

Frequency →

COMPARISON OF LOAD AND FRAGILITY

Loads And Strength

Deficient Regions

Load

Fragility

Frequency →
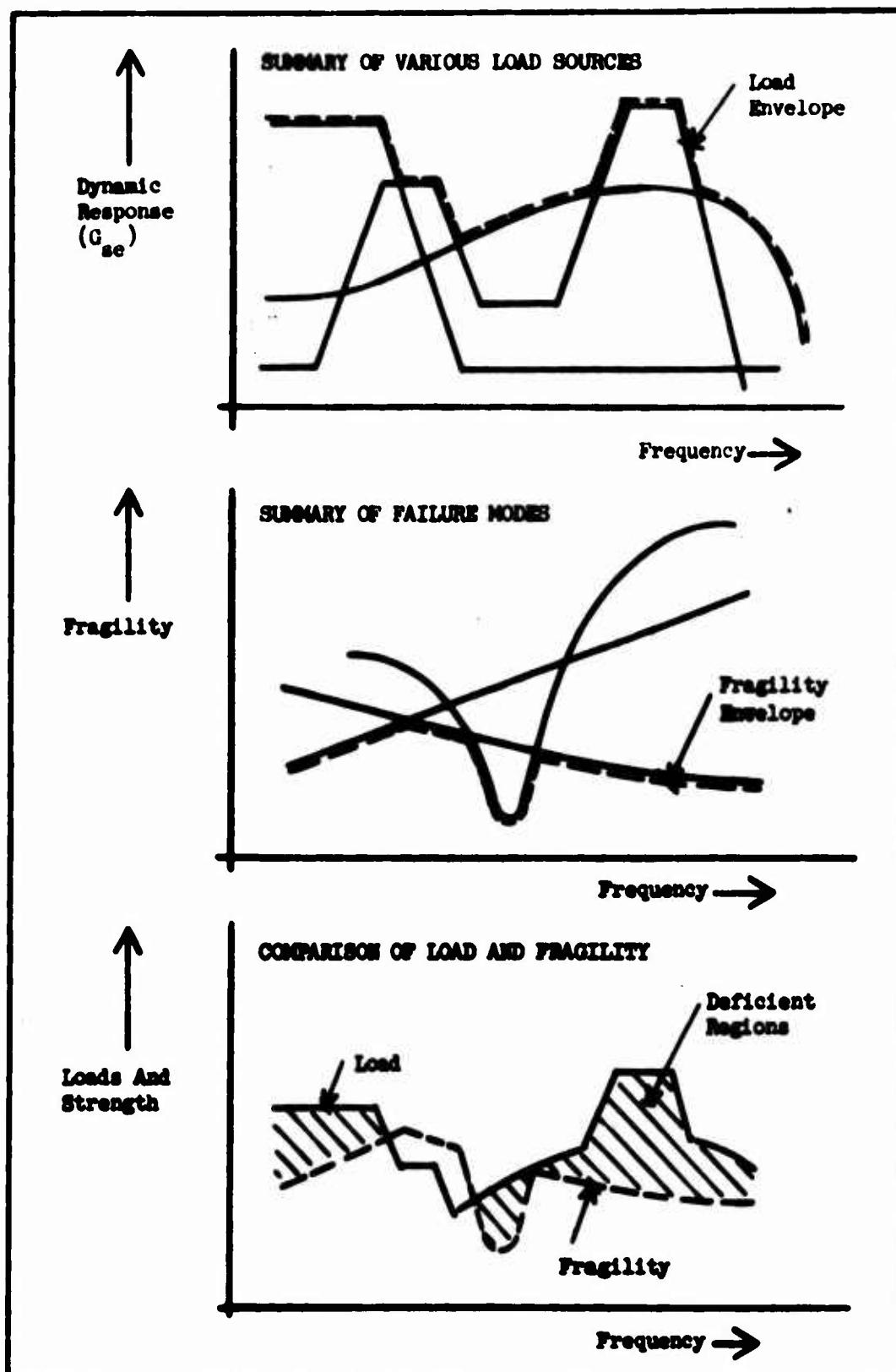
FRAGILITY AND LOADING CURVES: Comparison of envelopes indicates design deficiencies.

MARGIN OF SAFETY

The degree of adequacy of a part is determined quantitatively by comparing its
fragility to the applied load stresses, and is expressed as a margin of safety.

The margin of safety is a means of describing, quantitatively, the struc-
tural adequacy of an equipment element. There is no standard, universal,
definition of margin of safety, because different failure modes are
associated with different measures of strength and structural adequacy.
For example, fatigue failure is associated with both stress level and
number of repetitions. The margin of safety based on stress will be
numerically different from the margin of safety based on number of cycles.
This becomes immediately apparent when one considers the two different
methods for a situation in which the applied stress is below the endur-
ance limit. The margin based on life is infinite, while the margin based
on stress is some finite value.

In spite of the aforementioned difficulty, the following standard defini-
tion will be chosen for use in this Design Guide:

$$M.S. = \frac{allowable}{actual} -1 .$$

In this definition, "allowable" and "actual" refer to either load or
stress, depending on the mode of failure and the information available.
In the case of fatigue, margin of safety calculations will be based on
the comparison of stress with endurance limit, rather than considering
numbers of cycles. It is seen that if the margin of safety is positive
the design is adequate, and if the margin is negative the opposite is
true. It should be pointed out here that the desired value of safety
margin is not necessarily zero. Some designers choose to require that
the actual loads always be less than the allowable loads by some amount,
thus leading to a positive safety margin. Similarly, some designers
divide the strength of the material by a "safety factor" to establish the
allowable stress, or multiply the applied loads by a "safety factor" to
establish the design loads. The choice of these approaches is left to
the preferences of each individual user. For the purposes of the illus-
trations in this Design Guide, the "allowable" will correspond to the
true strength of the structure; the "actual" will correspond to the
dynamic loads developed in Section 3, and the desired value of margin of
safety will be zero.

This definition of margin of safety is especially useful in connection
with the fragility and loading curves discussed previously. The areas
of negative margin are those in which the fragility curve lies below the
loading curve; positive margins are opposite. Furthermore, the distance
between the curves is indicative of the value of the margin of safety.

The accompanying diagram illustrates two situations. At frequency f_1
the fragility is higher than the load so the margin of safety is positive.
At frequency f_2 the fragility is lower than the load so the margin of
safety is negative.

The magnitude of the margin of safety can be plotted on another graph,
also as a function of system natural frequency, as shown. This graph is
derived simply from the graphs of fragility and load versus frequency,
as indicated by the margin of safety formula. The advantage of presenting
this type of information is that the "good" and "bad" design regions
become immediately apparent.

Based on Life: $M.S. = \dfrac{N_1}{n_1} - 1 = \infty$

Based on Stress: $M.S. = \dfrac{S_2}{S_1} - 1 = \dfrac{S_2 - S_1}{S_1} \neq \infty$

At f_1: $M.S. = \dfrac{F_1}{L_1} - 1 = \dfrac{F_1 - L_1}{L_1} = $ Positive

At f_2: $M.S. = \dfrac{F_2}{L_2} - 1 = \dfrac{F_2 - L_2}{L_2} = $ Negative
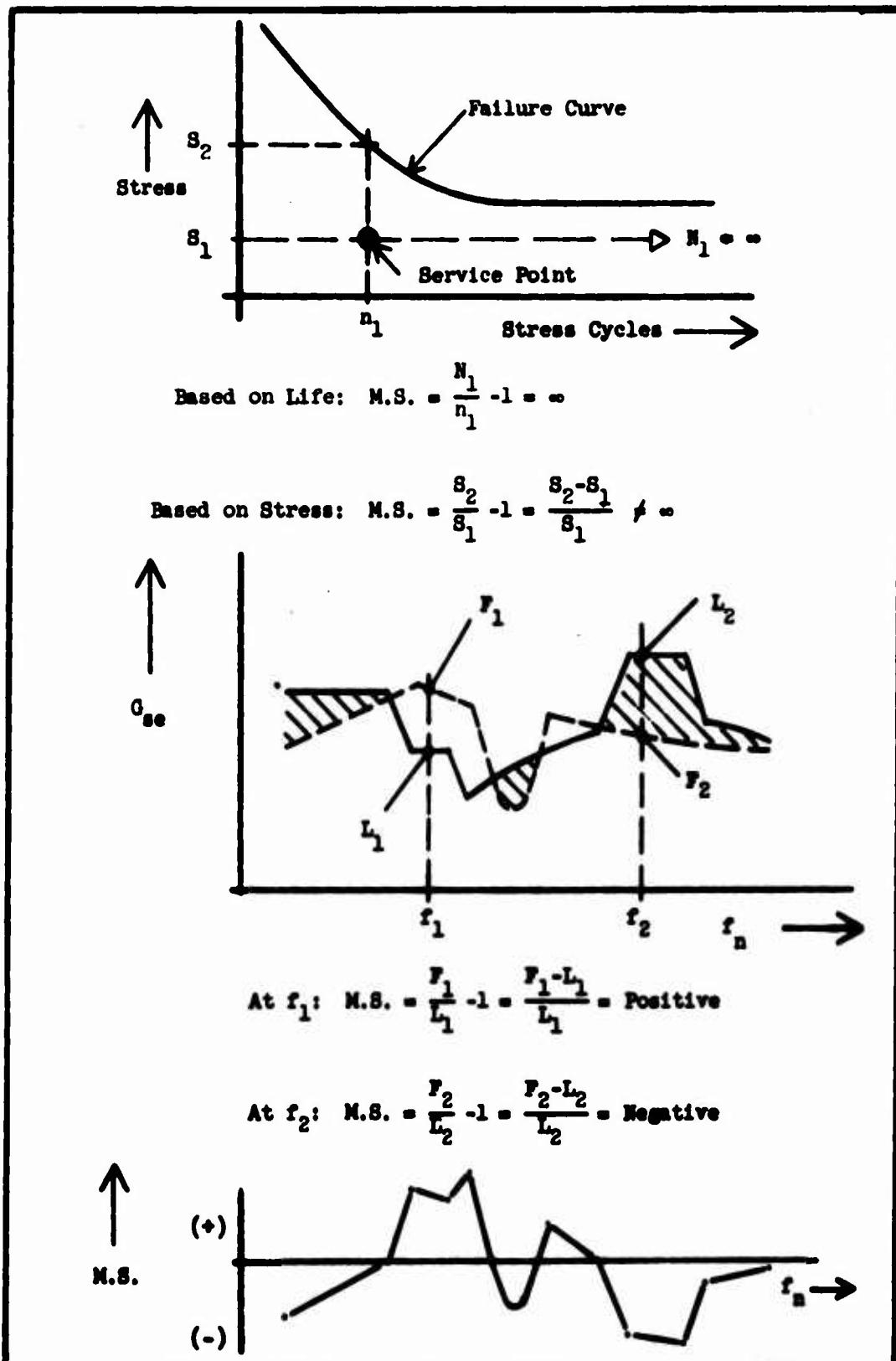
MARGIN OF SAFETY: Quantitative comparison of fragility and loading curves yield margins of safety at each natural frequency.

DETERMINATION OF CORRECTIVE ACTIONS

By examination of the fragility and loading curves, it can sometimes be concluded that corrective actions are possible.

The manner in which the margin of safety graph is displayed makes possible rapid evaluation of the structural adequacy of a design. Furthermore, it also provides insight into potential design changes which might be made as a means of improvement of the item and its reliability.

Two situations can arise in which design modifications are called for. The most obvious is the one in which the original design is found to be inadequate. The other involves a design which is adequate, but whose margin of safety can be enhanced relatively easily.

Design deficiencies are indicated by negative margins of safety, as noted previously. These occur when the fragility curve dips below the loading curve, as shown in the accompanying diagram. It is seen that the margin of safety of the design will be negative (that is, the design will be structurally deficient) if its resonant frequency lies between f_1 and f_2. The worst possible situation will result if the resonant frequency equals f_3.

There are a limited number of corrective actions available to the designer. The load curve cannot be altered because it is determined by environmental test requirements. (An exception is that the vibration loads could be reduced by the introduction of damping into the structure. However, for the purposes of this study, only nominal structural damping is considered, and no control over this parameter is anticipated.) However, the fragility curve can be altered by increasing the strength of the structure. This would lead to the situation indicated. The only other alternative is to modify the structure in such a way as to move its resonant frequency outside the critical range (to f_4 or f_5). Note that this implies not only that an increase in frequency would be beneficial, but also that a decrease in frequency could be equally beneficial.

The actual procedure to be followed in a particular case involves a series of steps. First a preliminary redesign is established on the basis of either strength or natural frequency. Then the new fragilities and natural frequencies are computed so that the corresponding margins of safety can be determined. This procedure will probably have to be repeated, perhaps several times, before the final design is attained.
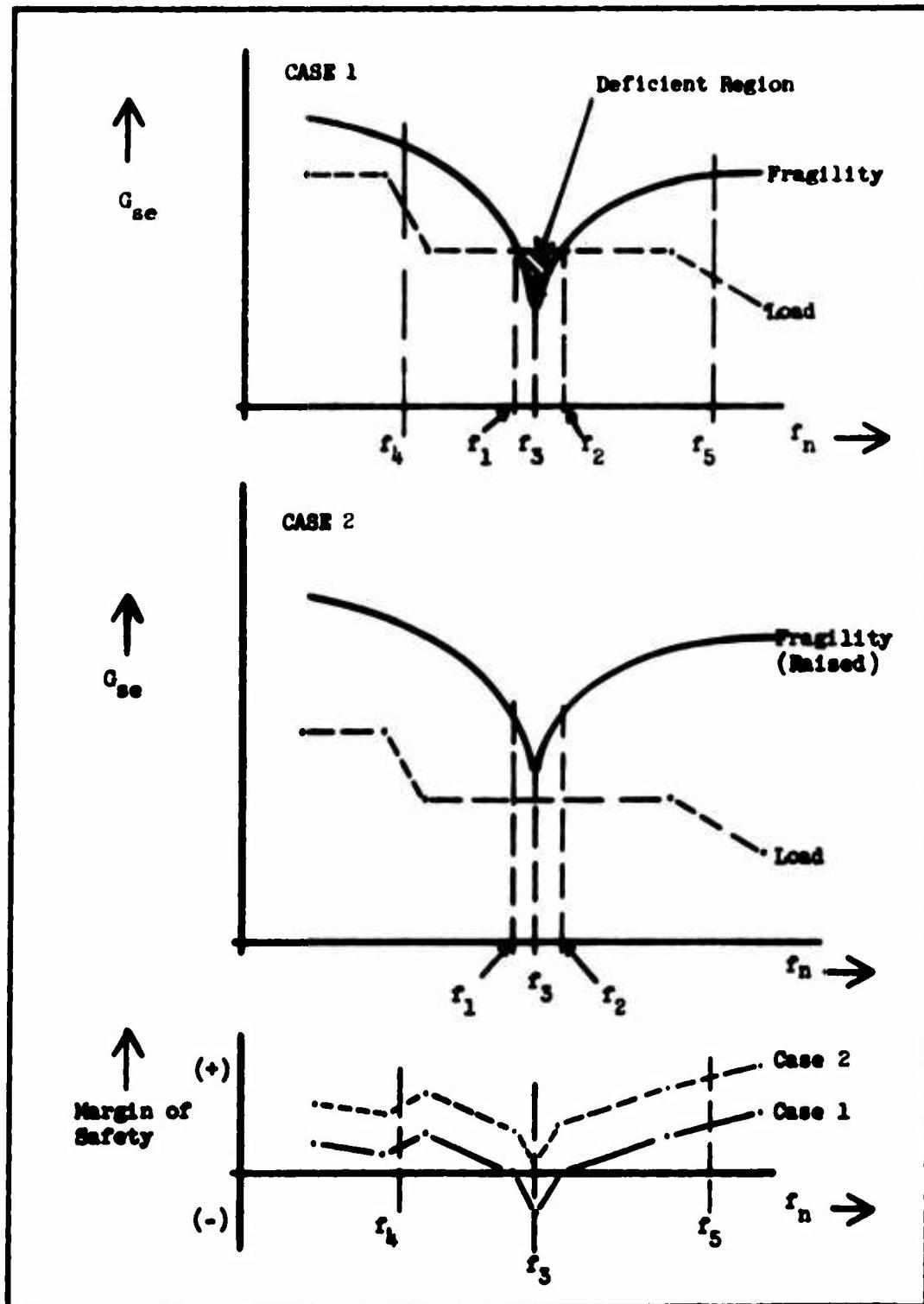
DESIGN EVALUATION CURVES: Corrective actions are based on the shape of the Margin-of-Safety graph.

VOLUME II

ANALYTICAL PROCEDURES

SECTION 6 — ILLUSTRATIVE EXAMPLE; EVALUATION OF A HYPOTHETICAL NEW DESIGN

- Procedural Applications
- Determination of Transfer Function
- Determination of Environmental Loading
- Determination of Failure Modes
- Computation of Fragilities
- Evaluation of Strength
- Corrective Actions
- Design Modifications

PROCEDURAL APPLICATIONS

Some illustrative examples will be used to demonstrate how the foregoing procedures are applied in practice.

A hypothetical design has been generated as a vehicle for demonstrating the design evaluation procedure. The item is described in the accompanying diagram. It has been specified as belonging to Class II: installed in an unarmored vehicle. This choice was made for two reasons. First, it is likely that most equipment falls in this class (and in Class III, which is subject to the same tests). Second, it is not subjected to ballistic shock testing, so that the design details may be influenced by more than one test (the ballistic shock is so severe that it overshadows all others, leading to a very special type of design). Certain design alternatives have been allowed to remain so that corrective actions may be illustrated at the end of the evaluation process.

The design will be carried, step by step, through the analytical procedures described in the preceding sections of this Design Guide. The various steps are identified in the flow diagram below:

Equipment Class ──────▶ Environment (Section 3)

Stress (Section 4)

Failure Modes (Section 4)

Redesigns (Section 5)

New Loads (Section 3)

Final Approval (Section 5)
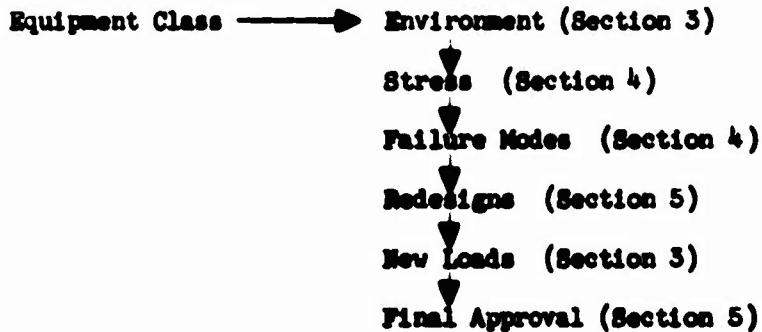
This approach is general, in the sense that it is the same for all types of equipment, regardless of the design details. In every case, the specification of "equipment class" leads to all the other steps.

Since this example is for illustrative purposes, it necessarily must be defined in terms of specific structural details. It must be recognized that any design which is encountered in practice will differ from the example in both gross and minor respects. It is beyond the scope of this Design Guide to attempt to catalogue the stress distributions and structural behavior of every possible combination of structural elements. Thus the user must possess a certain minimum understanding of structural analysis before being able to follow the steps in the approach advocated herein.

EQUIPMENT CONFIGURATION

$\frac{1}{16}$ x $\frac{1}{.}$ x $\frac{1}{2}$ Angles all Edges

0.032 Sheet
(5 Sides)

Item has
Resonant Freq-
uency of 250 Hz.
Allowable
Acceleration of
1000 g.

9

9

12

Main Platform
(1/4 in. Plate)

Total Weight = 75 lb.

Face Plate
(1/8 in. Plate

Distributed as follows:
   65.2 on Main Platform
   2.8 on Face Plate
   9.0 in Structure

Material: Aluminum Alloy 7075-T6 Throughout
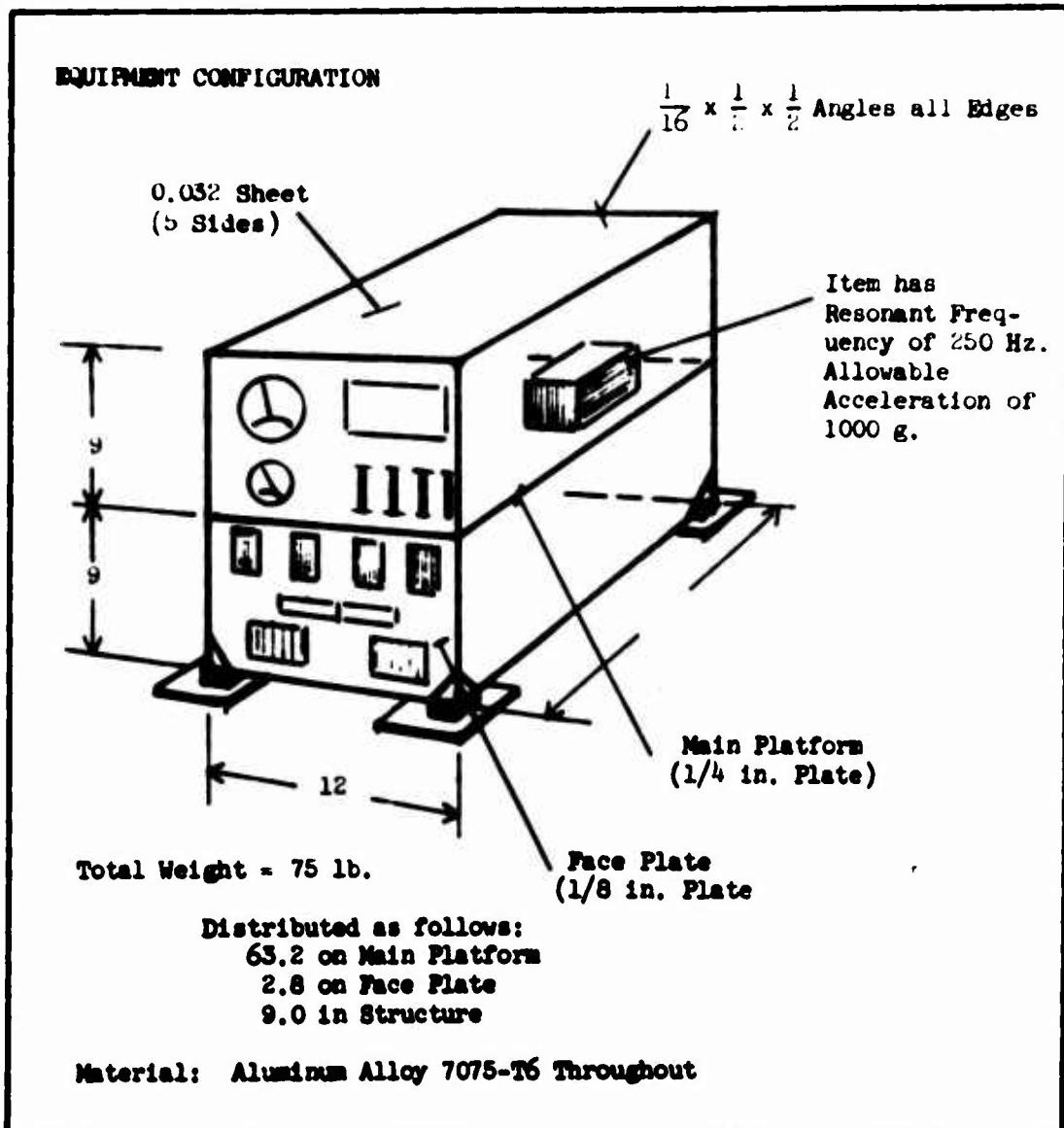
HYPOTHETICAL DESIGN: A 75-lb item of Class 2 Equipment is used to illustrate the analytical procedures.

DETERMINATION OF TRANSFER FUNCTION

The transfer function is synthesized from a computation of resonant frequency and an estimate of structural damping.

The transfer function for dynamic excitation is the one-degree-of-freedom sinusoidal amplification (transmissibility) function (see Section 3, Introduction). Two quantities are required to synthesize this function: natural frequency and damping.

To determine the natural frequencies, first consider the types of vibratory motion which the unit will exhibit. If the unit was bolted to a vibration test table, motions of the table in the directions of the principal axes (fore-aft, side-side, and vertical) would produce dynamic responses in those directions.

The motion in each of these directions can be interpreted as being represented by the response of a one-degree-of-freedom system. The corresponding natural frequency will depend on the stiffness characteristics of the structure relative to deflections in the directions of interest. The structural deformations associated with these directions can be envisioned by imagining that a static force is applied in each direction at the center of the main platform.

The natural frequency corresponding to the fore-aft direction is determined in accordance with the procedure presented in Section 3 (that is,

$$ f_n = \frac{3.13}{\sqrt{\Delta}}). $$

The static deflection, Δ, is the deflection due to a 75-lb. force (one-g) in the fore or aft direction. This force, applied at the platform location, is resisted by shear in the side plates, as indicated in the accompanying diagram. Only the lower half of each plate is considered, for the platform is supported at the center of the plate. Thus the resisting plates are 9 inches high, 24 inches wide, and 0.032 inches thick (each). The shear stress, strain, and deflection are computed as shown. The resulting natural frequency is seen to be 300 Hz.

The natural frequency in the fore-aft direction is determined in a similar fashion. The only difference is that the length of the plate is 12 inches instead of 24. The plate thicknesses are the same, assuming that the perforated 1/8-inch front plate is equivalent to a uniform plate 0.032 inches thick. The resulting calculations lead to a natural frequency of 210 Hz.

The natural frequency for vertical motion is determined in a somewhat different manner. Here the one-g force is assumed to be uniformly distributed over the main platform. This approach is based on the observation that the platform provides the major flexibility in the vertical direction, since it is loaded in bending while the side plates carry the vertical loading in direct tension/compression. The static deflection is taken to be the deflection at the center of the plate under the action of the weight supported by the platform (63.2 lb). This deflection is

computed as shown in the accompanying diagram. It is seen that the
natural frequency is found to be 55 Hz. The three undamped (and uncoupled)
transmissibility curves, shown following, are computed from the relationship

$$TR \quad \frac{1}{1 - \left(\frac{f}{f_n}\right)^2}$$

Regarding damping, it is assumed that the structure is assembled by means
of threaded fasteners or rivets. Thus the damping level will be in the
vicinity of 5 percent of critical. The resulting maximum amplification
factor will be 10 at each resonance point. The corresponding truncated
transmissibility curves are shown in the accompanying figure. These are
the transfer functions for excitation in each of the three principal
directions. (See Page 5-14)
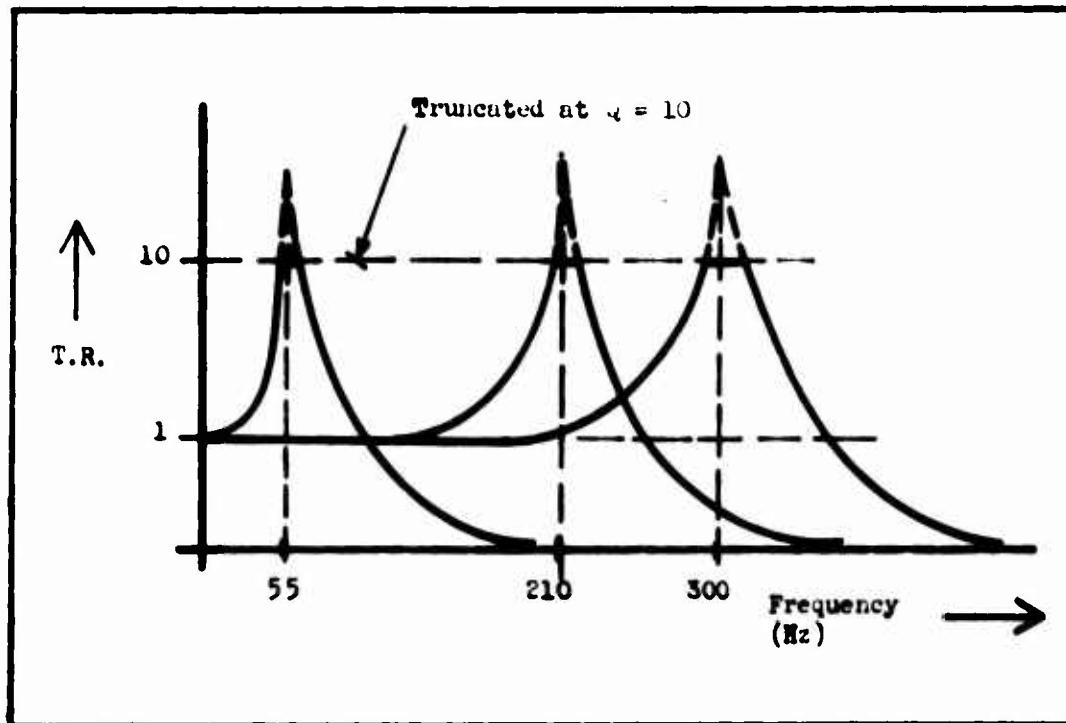
TRANSFER CHARACTERISTICS: The transmissibility (as a function of the
frequency of disturbance) is idealized in the response curves for the
three principal directions.

DETERMINATION OF TRANSFER FUNCTION (Continued)

FORE-AFT MOTION
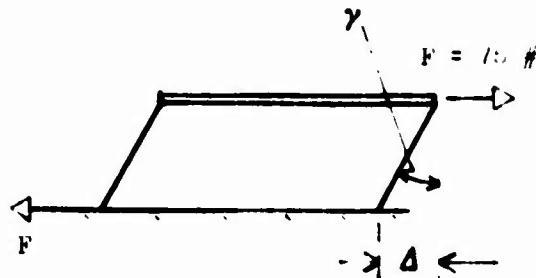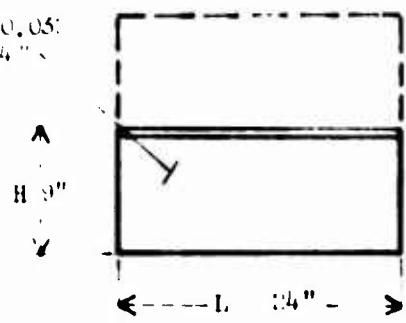
Shear Stress $\tau = \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{Lt} = \dfrac{75}{24 \times 0.064} = 48.8$ psi (Ref. 5)

Shear Strain $\gamma = \arctan \dfrac{\Delta}{H} \cong \dfrac{\Delta}{H} = \dfrac{\Delta}{9}$ (Ref. 5)

Shear Modulus $G = \dfrac{\text{stress}}{\text{strain}} = \dfrac{48.8}{(\Delta/9)} = \dfrac{439.2}{\Delta}$

Solve for Δ using $G = 4 \times 10^6$ for aluminum:

$$\Delta = \frac{439.2}{G} = \frac{439.2}{4 \times 10^6} = 109.8 \times 10^{-6} \text{ inches}$$

Natural Frequency: $f_n = \dfrac{3.13}{\sqrt{\Delta}} = \dfrac{3.13}{\sqrt{109.8 \times 10^{-6}}} =$

$$\frac{3.13}{10.47 \times 10^{-3}} = 300 \text{ Hz}$$

SIDE-SIDE MOTION
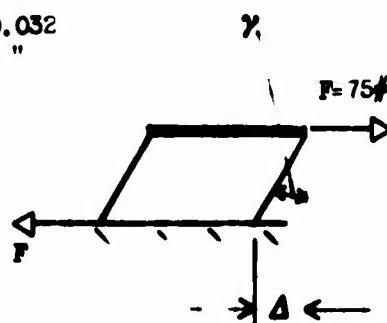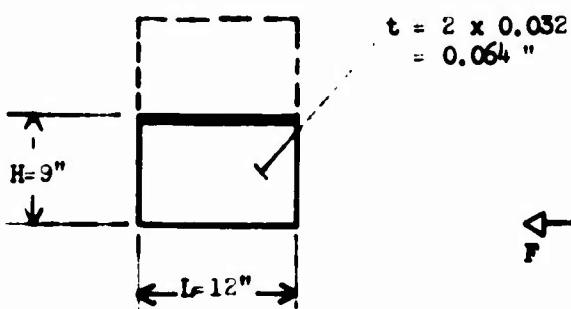
Shear Stress $\tau = \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{Lt} = \dfrac{F}{1 \times 0.054} = 97.5$ psi

Shear Strain $\gamma = \dfrac{\Delta}{H} = \dfrac{\Delta}{9}$

Shear Modulus $G = \dfrac{97.5}{(\Delta/9)} = \dfrac{877.5}{\Delta}$

$$\Delta = \dfrac{877.5}{G} = \dfrac{877.5}{4 \times 10^6} = 219.4 \times 10^{-6}$$

Natural
Frequency: $f_n = \dfrac{3.13}{\sqrt{\Delta}} = \dfrac{3.13}{\sqrt{219.4 \times 10^{-6}}} = \dfrac{3.13}{14.8 \times 10^{-3}} = 210$ Hz

VERTICAL MOTION
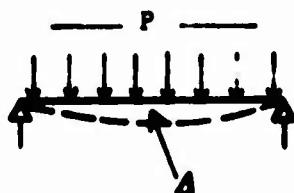
Main Platform Idealized as
Simply-Supported Plate
(width = 24, length = 12,
thickness = 1/4)

Static Deflection $\Delta = 0.111 \dfrac{Pb^4}{Et^3}$

where

P = Pressure = force/area = $\dfrac{63.2}{24 \times 12}$ = 0.22 psi

b = Length = 12 inches

E = Modulus of Elasticity = 10×10^6 psi

t = Thickness = 0.25 inches

$$\Delta = 0.111 \times \dfrac{0.22 \times (12)^4}{10 \times 10^6 \times (0.25)^3} = \dfrac{0.0506 \times 10^4}{0.156 \times 10^6} = 0.00325 \text{ in.}$$

Natural
Frequency: $f_n = \dfrac{3.13}{\sqrt{\Delta}} = \dfrac{3.13}{\sqrt{0.00325}} = \dfrac{3.13}{0.057} = 55$ Hz

DETERMINATION OF ENVIRONMENTAL LOADINGS

The dynamic loadings, or responses of the equipment, are determined on the basis of equipment class, as shown in Section 3.

The equipment under consideration is assumed to belong to Class II: installed in an unarmored vehicle and operated when the vehicle is not in motion. As such, the required quality assurance tests are:

Vibration (10-55 Hz)
Vehicular bounce
Bench handling shock
Drop shock   } Detailed on Page 2-3
Munson road course
Perryman road course
Railroad humping test

The vibration test is a resonance survey, and is not considered important for structural design purposes. The dynamic environments for the remaining tests are shown in the accompanying diagram, as determined in Section 3 of this volume. The shock curve corresponds to drop tests from a height of 48 inches, as prescribed for equipment which weighs less than 100 lbs. The random vibration curve corresponds to the vehicular bounce test.

The anticipated responses for the item in question are determined from the accompanying graphs via the resonant frequencies determined previously: 55 Hz, 210 Hz and 300 Hz. The shock responses are 180g, 660g, and 950g. The random vibration responses (3σ) are 50g, 96g, and 112g.
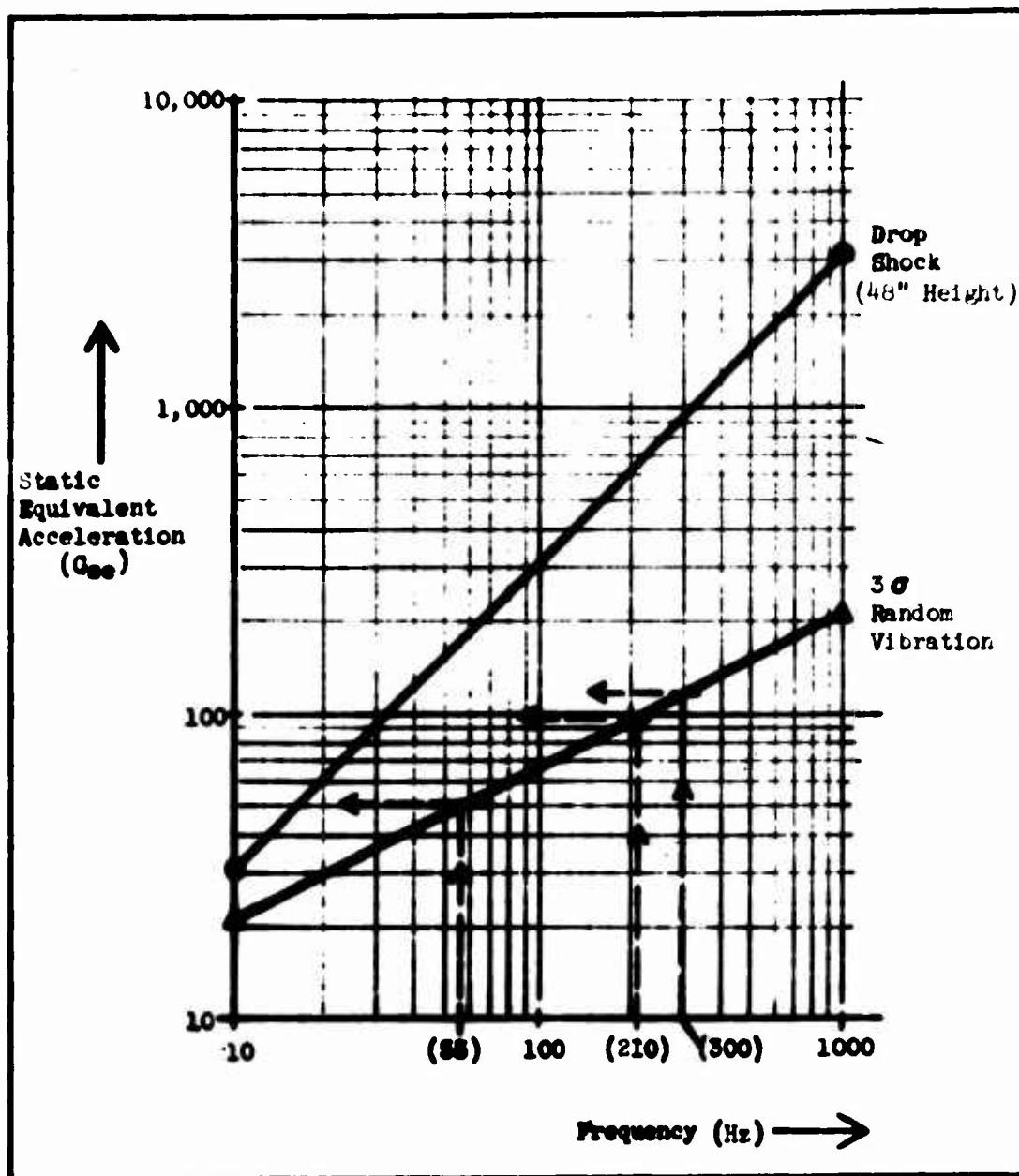
DYNAMIC RESPONSES: The static Equivalent Acceleration for the hypothetical Class II example are read at 55, 210 and 300 Hz as 50 g, 96 g, and 112 g respectively.

DETERMINATION OF FAILURE MODES

The failure modes to be anticipated are determined by inspection of the design and consideration of its structural behavior.

Inspection of the structural nature of the hypothetical design reveals that the primary structural failure modes will be as follows:

1. Shear buckling of the side and back panels due to side-side and fore-aft loads.

2. Bending failure (yield or fatigue) of the main equipment platform due to vertical loads.

3. Axial failure (yield or fatigue) of the vertical frame members due to horizontal and vertical loads.

4. Excessive deformation.

5. Malfunction of the 250 Hz item whose fragility is 1000 g.

The internal loads and stresses are determined by analysis, as follows:

1. Panel shear stress:

   Fore-aft and side-side dynamic responses of the equipment will produce shear stresses in the side and rear panels. The highest stresses will occur in the rear panel because its length (and corresponding shear area) is least. The shear force on the rear panel is half of the dynamic load, the other half being carried by the thick front panel. The resulting stress is the force (1/2 x weight x dynamic load) divided by the shear area (length x thickness). In terms of static equivalent acceleration:

$$\tau = \frac{\frac{1}{2} \times \text{weight} \times G_{se}}{b \times t} = \frac{\frac{1}{2} \times 75 \times G_{se}}{12 \times 0.032} = 97.5 \ G_{se} \ \text{psi}$$

Thickness $t = 0.032$

$F = 1/2 \times wt \times G_{se}$

9

Shear Area $= 12 \times 0.032$

12
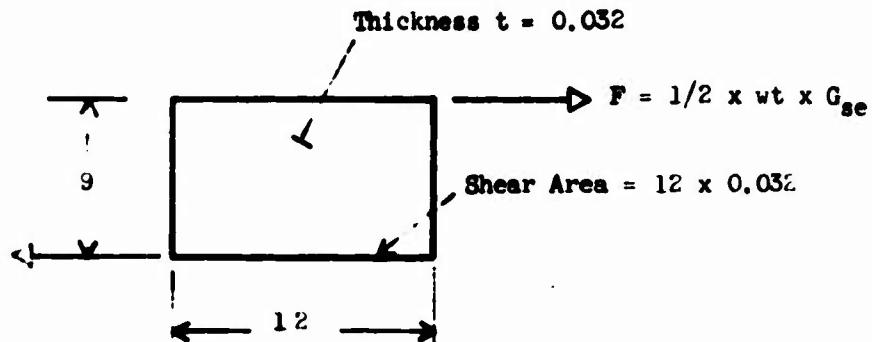
2. Bending of main equipment platform:

The main equipment platform is subjected to vertical loads which cause bending stresses in the platform. The loading is assumed to be uniformly distributed, and results from vertical shock and vibration response. The maximum stress in a simply supported plate due to uniform pressure is:[8]

$$S_{max} = \beta \frac{Pb^2}{t^2}$$

where P is the distributed pressure and β is a constant depending on the ratio a/b (a = length and b = width of the plate).

For this case, where a/b is 24/12 = 2.0, the constant is β = 0.61, so that the stress is:

$$S_{max} = 0.61 \frac{P(12)^2}{(0.25)^2} = 1400 \text{ P (psi)}$$

The pressure P is computed from the weight of equipment supported by the platform and the static equivalent acceleration:

$$P = \frac{\text{weight} \times G_{se}}{\text{area}}$$

$$P = \frac{63.2 \times G_{se}}{12 \times 24} = 0.219 \, G_{se} \text{(psi)}$$

Thus the stress is:

$$S_{max} = 1400 \times 0.219 \, G_{se} = 306 \, G_{se} \text{(psi)}$$

DETERMINATION OF FAILURE MODES (Continued)

3. Axial failure of main frame:

The four tie-down points carry all the vertical support loads through the main frame members. As a result, both fatigue and yielding failures could result. The force carried by each vertical member due to vertical loading is one-quarter of the total force.

$$F = \frac{1}{4} \times 75 \times G_{se} = 18.75 \ G_{se}$$

However, this is smaller than the force due to lateral loading:

$$2F \times 12 = 9 \times 75 \times G_{se}$$

$$F = 28 \ G_{se}$$
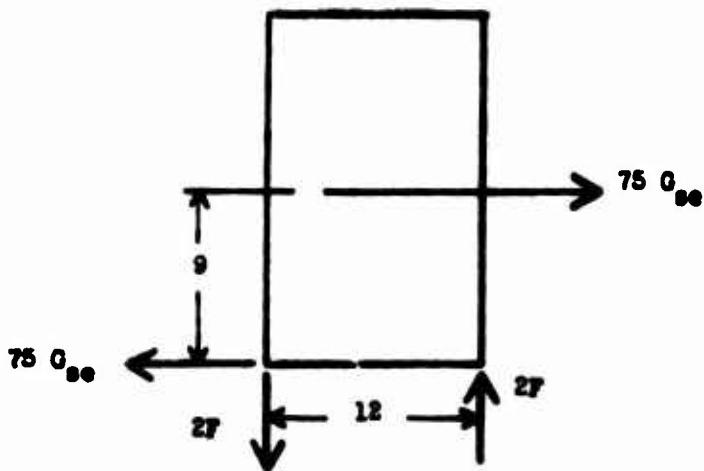
4. Excessive deformation:

The failure mode corresponding to excessive deformation involves the lateral displacement of the equipment. This is related to acceleration and frequency by the formula:

$$\text{acceleration} = \text{displacement} \times (\text{frequency})^2$$

$$G_{se} \times 386 = \delta \times (2\pi f)^2$$

$$\delta = \frac{386 \, G_{se}}{4\pi^2 f^2} = 10 \, \frac{G_{se}}{f^2}$$

Thus, knowing the acceleration and the frequency, the displacement is readily determined.

5. Component malfunction:

The response of the fragile component depends on the input to it and its transmissibility characteristics. If it is assumed that the component behaves as a one-degree-of-freedom system with a maximum amplification of 10, its response to an input of G_{se} will be as shown in the accompanying diagram. This is interpreted to mean that the response of the structure is the input to the component.
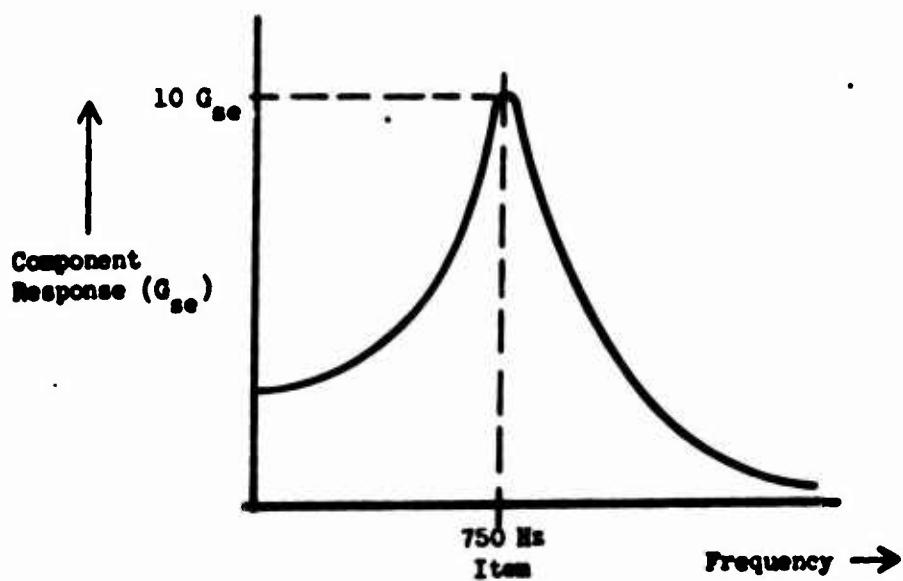
COMPUTATION OF FRAGILITIES

Classical stress analysis procedures are used to compute the fragilities corresponding to the anticipated modes of failure.

The fragilities corresponding to the anticipated modes of failure depend on the allowable stresses, as governed by the structural material (7075-T6 aluminum) and configuration. Typical values of the material properties are:

Modulus of Elasticity (E) = 10×10^6 psi

Yield Strength = 70,000 psi

Fatigue Strength (10^6 cycles) = 20,000 psi

These properties are used to determine the fragilities (that is, allowable values of G_{se}) as follows:

1. <u>Shear buckling</u> (Ref. 10, Pg. 359): The critical shear stress for a simply supported rectangular plate, as shown, is:

$$S_{cr} = K \frac{\pi^2 D}{b^2 t}$$ Smaller Dimension

where $D = \frac{Et^3}{12(1-\mu^2)}$
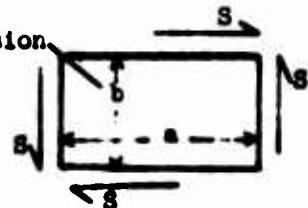
and the value of K depends on the ratio a/b. A limiting minimum value which can be used with assurance of conservatism is K = 5. Thus the critical shear stress is

$$S_{cr} = 5 \frac{\pi^2}{b^2 t} \frac{Et^3}{12(1-\mu^2)} = 4.64 \frac{Et^2}{b^2}$$

where Poisson's ratio is assumed to be $\mu = 1/3$. For the case in point, the largest dimension b is b = 9 inches, (all panels).

Thus: $S_{cr} = 4.64 \frac{10 \times 10^6 (0.032)^2}{(9)^2} = 585$ psi

Equating the critical and actual stresses:

97.5 G_{se} = 585, G_{se} = 585/97.5 = 6

Thus 6 g is the fragility level, independent of frequency, corresponding to the shear buckling mode of failure.

2. Platform bending: Both yield and fatigue must be considered as potential failures of the main platform. The corresponding allowable stresses are 70,000 psi for yield and 20,000/1.8 = 11,000 psi for fatigue (assuming a stress concentration factor of 1.8 for bending of a plate containing a hole).[11] The corresponding fragilities are determined by equating actual and allowable stresses:

Yield: 306 G_{se} = 70,000;

        G_{se} = 70,000/306 = 230 g

Fatigue: 306 G_{se} = 11,000

        G_{se} = 11,000/306 = 36 g

3. **Main frame:** The yield and fatigue allowable stresses for the main frame are 70,000 psi and 20,000/3 = 6700 psi (assuming a stress concentration factor of 3 due to rivet holes). Thus the fragilities are:

Yield: $448 \, G_{se} = 70,000;$ $G_{se} = 70,000/448 \quad 156 \, g$

Fatigue: $448 \, G_{se} = 6700;$ $G_{se} = 6,700/448 \quad 15 \, g$

4. **Excessive deformation:** The lateral displacement of the equipment is assumed to be limited to a maximum of 0.5 inches by neighboring equipment. The corresponding accelerations are determined as follows: (See formula on Page 6-11, where $\delta = 0.5$.)

$$0.5 = 10 \frac{G_{se}}{f^2} \; ; \quad G_{se} = 0.05 \, f^2 \; (g's).$$

Thus the fragility is a function of frequency, and is plotted as such on the accompanying graph.

5. **Component malfunction:** The allowable input to the fragile component depends on its transmissibility and its own fragility. Since it can withstand an acceleration level of 1,000 g, the allowable input is 100 g at 250 Hz (where the transmissibilty is 10) and is 1000 g at low frequency (where the transmissibility approaches 1). Thus the fragility varies with frequency, and is given by the expression:

$$G_{se} = \frac{1000 \, g}{TR}$$

where TR is the transmissiblity of the component as shown in the preceding topic. The fragility is shown in the accompanying diagram.

The complete set of fragility curves is shown in the accompanying diagram. There is thus displayed a picture of the allowable dynamic responses as a function of the resonant frequency of the equipment.
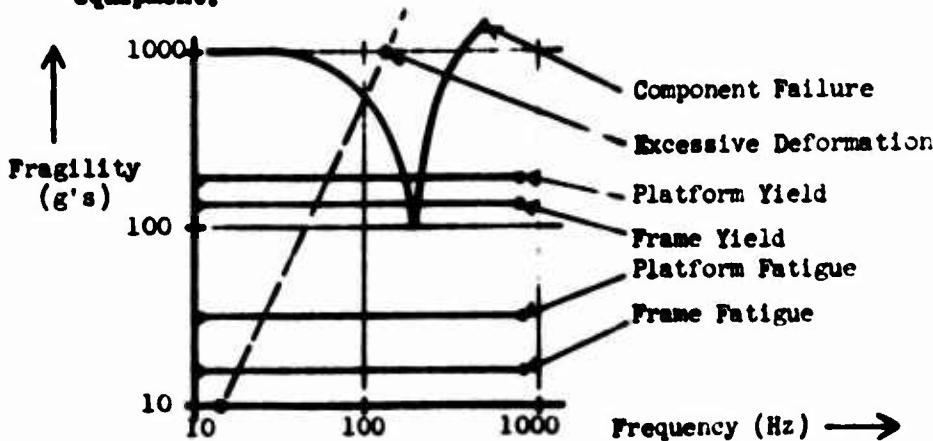
EVALUATION OF STRENGTH

Comparison of the fragility and response curves shows whether or not the design is adequate.

The accompanying diagrams show the previously developed fragility and loading graphs. Here the shock fragility and shock loading curves have been plotted on one graph, while those corresponding to vibration are on the other. Below each of these graphs is shown a graph of safety margin as a function of natural frequency. Also indicated are the three basic natural frequencies of this item of equipment: 55 Hz, 210 Hz, and 300 Hz.

It is apparent that the margins of safety are negative at all three natural frequencies, thus indicating that the proposed design is inadequate. Further, there are no frequencies at which the margin of safety is positive. This indicates that major redesign will probably be required, since the structural adequacy cannot be sufficiently enhanced by simply shifting natural frequencies. The corrective actions to be taken are discussed in the next topic.

The particular format used here for displaying the fragility and loading information is especially useful for evaluating corrective actions. Separating the shock and vibration graphs allows the fatigue and other modes of failure to be evaluated independently. Thus only the worst case need be corrected, because the others will then be improved automatically (for the same structural element). It should also be noted that, while only the envelope of loads is required, the separate graphs for the various failure modes must be presented. An envelope of failure modes, while useful in computing safety margin, does not provide sufficient information for redesign purposes.
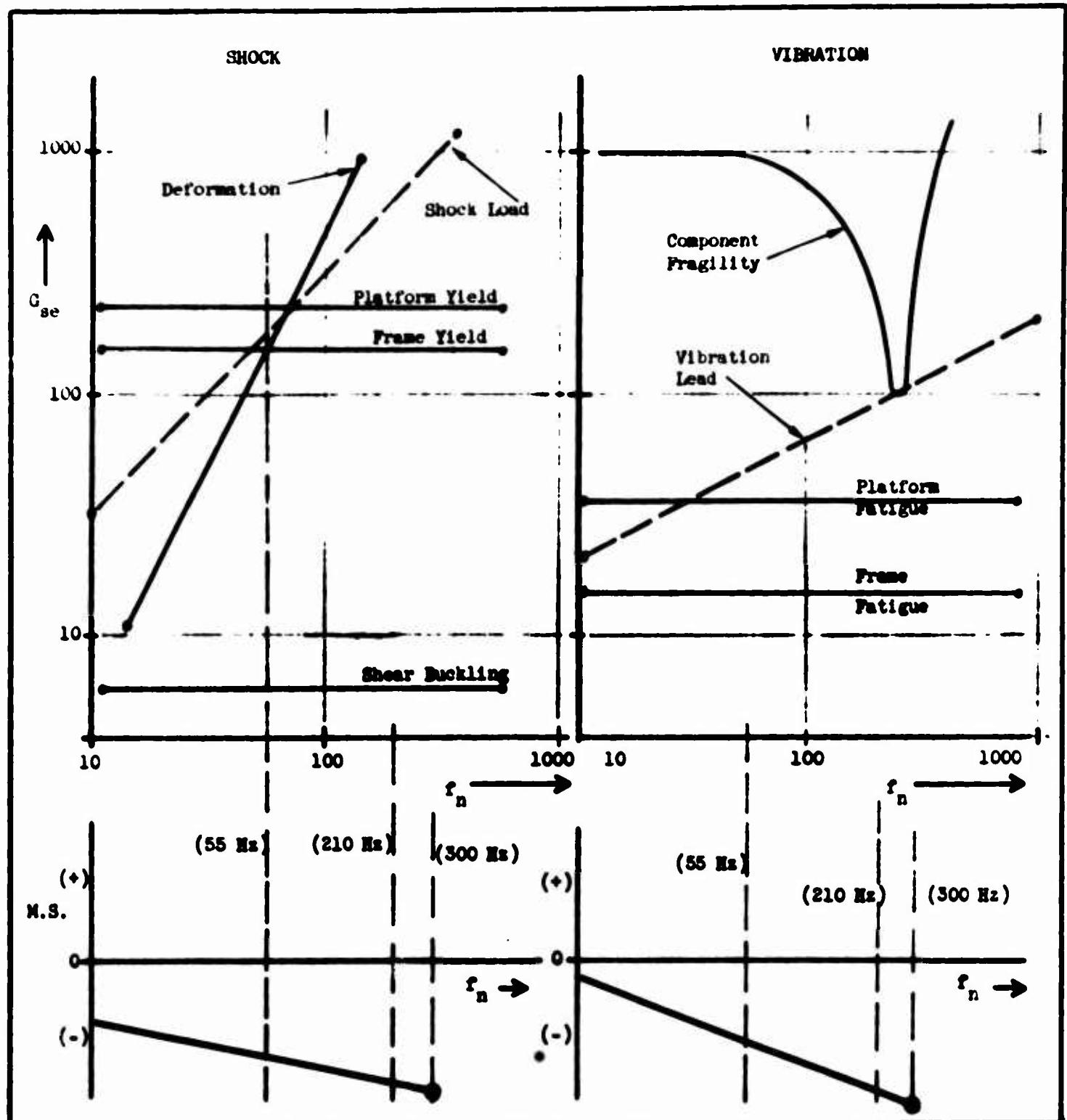
COMPARISON OF FRAGILITY AND LOADING CURVES: Negative Margins of safety at natural frequencies indicate design is not adequate.

CORRECTIVE ACTIONS

It can be seen that certain design changes are required to implement a sufficient improvement in dynamic strength.

It has been shown that the design, as proposed, is inadequate. In order to determine how the design may be changed to increase its adequacy, it is essential to consult the fragility graphs. Here the separate fragility graphs, not just the envelope, should be considered. It is sufficient to make a comparison with the loading graph envelope, however. This comes about because the fragility curves will change shape as the design changes, while the loading curves are independent of the design details.

It is seen that the most severe mode of failure (corresponding to greatest negative safety margin) is panel shear buckling, followed by frame fatigue.

On the basis of the foregoing, it can be concluded that certain changes can be made which will have a great effect on the structural adequacy of the equipment. The changes fall into two major categories: shifts in natural frequency and increases in fragility levels.

Consider the benefits of changing natural frequency. If the frequencies f_2 and f_3 can be shifted sufficiently far, the mode of failure corresponding to component malfunction can be precluded. Note that the shift can be either higher or lower. In the interest of saving weight, the frequencies should be shifted downward, if possible. Referring to the calculation of $f_2 = 210$ Hz., it is seen that the determing factor is the shear stiffness of the cabinet panel. Since this item is already critical in buckling, it must be thickened rather than thinned. Thus the frequency f_2 will be increased rather than decreased (unless the configuration is changed). The frequency increase will be accompanied by an increase in buckling strength of the cabinet panel. It can also be seen that there is no advantage to simply shifting the frequency f_1 because the loading and fragility curves are approximately parallel in those region of $f = 55$ Hz.

Inspection of the individual fragility curves indicates that in order to raise the overall fragility levels the following modes of failure must be prevented: shear buckling of panels, main platform fatigue, frame fatigue, frame yield, and platform yield. These corrections can be accomplished by changing the material thicknesses appropriately. Examination of the loading curves indicates that the governing buckling and yield load (the drop shock test), is 1000g in the vicinity of 300 Hz. The governing fatigue load (random vibration), is 100g at the same natural frequency. Thus, the fragilities should be increased to these levels for preliminary redesign. Detailed reevaluation can then follow. Each mode of failure is discussed individually in the following topic.
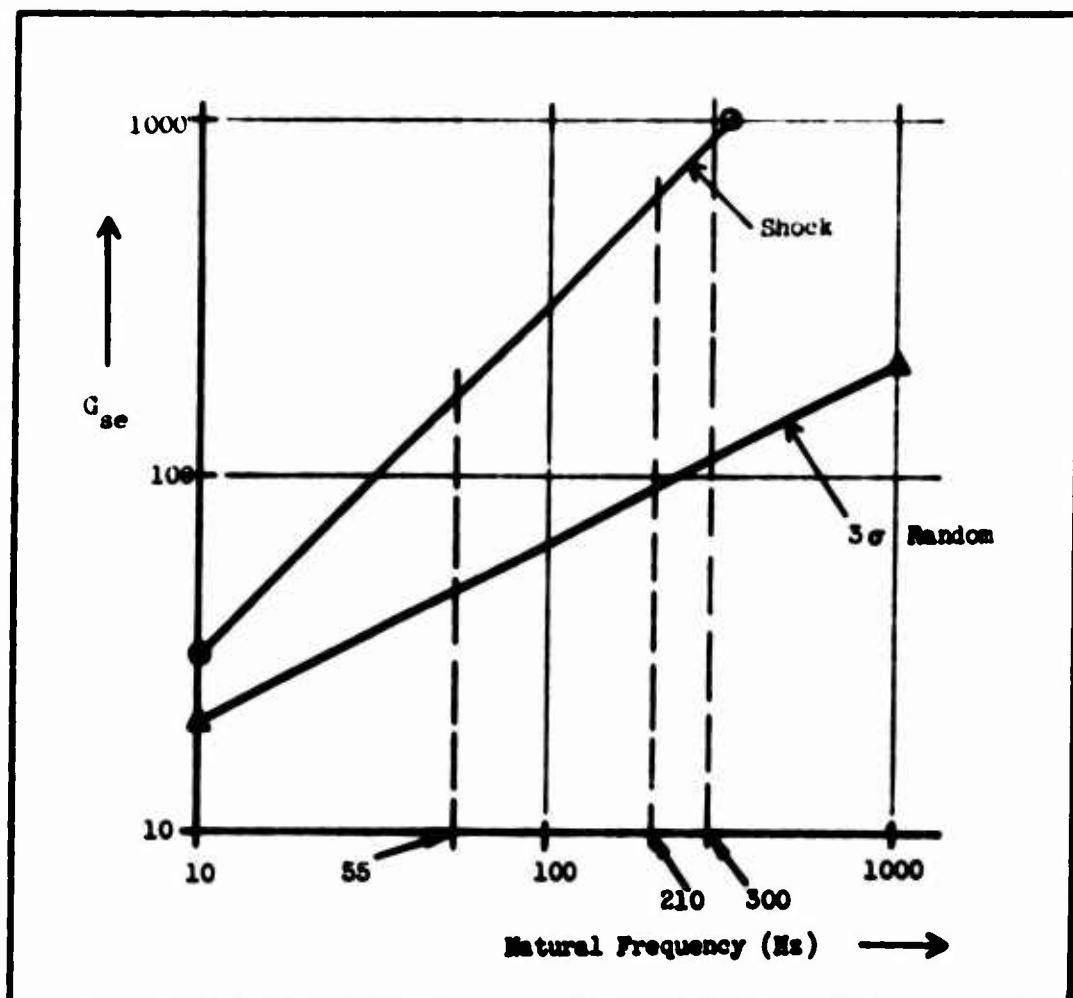
NEW DESIGN LOADS: Design changes lead to changes in loads - new f_n assumed to be 300 Hz.

DESIGN MODIFICATIONS

The strength calculations are used to establish new values of structural parameters until the design is shown to be adequate in all respects.

The buckling of the side and back cabinet panels is governed by the factor $(t/b)^2$ as shown previously. In order to raise the fragility from 6g to 1000g, the ratio h/b must increase by a factor of $\sqrt{1000/6} \cong 13$. This can be accomplished by introducing ribs to decrease b from 9 inches to 3 inches and thickening the panel so that h increases from .032 to .125 inches. The resulting panel is inordinately thick (and heavy) which indicates that in this case it would appear wiser to change the design configuration than to modify the existing one. This is accomplished by introducing diagonal members to support the side loads. Then buckling will be resisted by the frame members rather than the panels. The required sizes are determined as follows. The maximum side load of 1000g (shock test with $f_n = 300$ Hz) produces a force of $1000 \times 75 = 75,000$ lb. (of which only half is carried by the back face). The most heavily loaded diagonal member is the tension member across the back (neglecting the carrying capacity of the compression member). The load it carries (see diagram) is 46,900 lb. To prevent tensile failure (tensile strength = 80,000 psi) a cross section of $46,900/80,000 = 0.59$ sq. in. is required. For simplicity's sake, this can be considered typical of all eight diagonal members. The vertical frame member is subjected to compressive loads of 28,200 lb. as shown. To prevent buckling, the required moment of inertia is:

$$ I = \frac{PL^2}{\pi^2 E} = \frac{28,200 \times (9)^2}{\pi^2 \times 10 \times 10^6} = .0231 \text{ in}^4 \text{ (for an Euler column).} $$
$$ \text{(See Reference 8)} $$

This can be provided by a 1-1/8 x 1-1/8 x 1/8 angle, with a moment of inertia of .030 in^4 and an area of .27 in^2. However, an area of .40 in^2 is required to prevent yielding (yield strength = 70,000 psi). Thus a 1-1/2 x 1-1/2 x 3/16 angle is required (area = .53).[12]

The fatigue modes of failure for the original frame and platform can be precluded by properly reducing the stress levels in those elements. Considering the fatigue load to be 100g (3σ random at 300 cps) and the existing fragilities to be 36g for the platform and 15g for the frame, the stresses in those elements must be reduced by factors of $100/36 = 2.8$ and $100/15 = 6.7$ respectively. The platform stresses vary with the square of the thickness (as in bending of a beam), so that the 1/4 inch plate must be thickened by a factor of $\sqrt{2.8} = 1.7$.

The resulting thickness is 0.40 inches, which increases the yield fragility to $230 \text{ g} \times (.4/.25)^2 = 600 \text{ g}$. This is not sufficient to meet the 1000g requirement, which leads to a thickness of

$$ .25 \text{ in.} \left(\frac{1000 \text{ g}}{230 \text{ g}}\right)^2 = .25 \times 2.1 = \underline{.525 \text{ inches.}} $$

The fatigue fragility is then increased to $36 \text{ g} (.525/.25)^2 = 160 \text{ g}$.

The frame stress varies directly with cross-sectional area, which thus must increase to $1/16 \times 6.7 = .42$ in^2. This area is exceeded by that of the angles provided to prevent buckling. Thus, fatigue of the frame is not a critical failure mode.

The result of all these corrective actions is a new design which, at first, is only preliminary. The next step is to make the corresponding changes in the fragility curves and natural frequency calculations, in order to evaluate the new design, using the same approach as the original design. This entire process is iterated until the final configuration is found to be adequate in all respects.

If this process were to be carried through for this illustrative example, the resulting design would be as indicated in the accompanying figure.

Platform 0.46" Thick

(4) Vertical Frame Members
(1-1/2 x 1-1/2 x 1/8 angles)

(8) Diagonal Members
(0.47 in.^{2} cross-section)

37,500 On Back Side

75,000 Total at C.G. Location

F_1 = 37,500 (15/12) = 46,900 lb

F_2 = 37,500 (9/12) = 28,200 lb

Fore-Aft: f_n = 135 Hz, Side-Side: f_n = 150 Hz, Vertical: f_n = 75 Hz

Margins of Safety: (Reflecting design modification)

Frame Buckling:	1150/450 -1 = 1.5
Frame Yield:	890/450 -1 = 1.0
Frame Fatigue:	84/80 -1 = 0.05
Diagonal Yield:	700/450 -1 = 0.55
Diagonal Fatigue:	80/80 -1 = 0
Platform Yield:	584/225 -1 = 1.6
Platform Fatigue:	92.5/56 -1 = 0.65
Equipment Malfunction:	600/450 -1 = 0.33
Excessive Deformation:	275/225 -1 = 0.22

FINAL DESIGN: Modifications are iterated until the desired safety margins are achieved.

VOLUME II

ANALYTICAL PROCEDURES

SECTION 7 - APPENDIX

- Bibliography

- Glossary

- Symbology

- Drop Test Shock Response

- Bench Handling Shock Response

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GLOSSARY

Certain terms must be defined in order to allow intelligent discussion of the process of design evaluation. This is necessary because a given term might be defined in a number of different ways, depending on the background of the definer. The expressions of major interest here are the following:

Equipment Class — The designation referring to the type of service to which a piece of equipment will be subjected.

Quality Assurance Tests — The tests required by the user to show that the equipment is sufficiently strong to survive in service.

Proposed Design — The structural configuration to be evaluated by the process discussed herein. This may be the original proposal or a modification thereof.

Transfer Function — The quantitative description of a system's dynamic characteristics which relates the dynamic inputs to the dynamic responses.

Input — The dynamic motions imposed on a system by the quality assurance tests.

Response — The dynamic motions of the system resulting from the test inputs.

Fragility — The maximum dynamic response to which a system should be exposed, based on the strength of the equipment.

Margin of Safety — The quantitative measure of the strength of a part, relating actual stresses to allowable stresses.

Redesign — The process of design modification which results in new responses and stresses.

Fatigue — Failure caused by repeated application of stresses below the static strength of the material.

Cumulative Damage — The degradation of fatigue strength caused by exposure to a fatigue environment.

Power Spectral Density — The quantitative description of a random motion which indicates the frequency distribution of the energy content.

Resonance — The dynamic phenomenon in which the system amplifies the input motions.

Damping — A mechanism for removing energy from a dynamic system, thus preventing infinite responses at resonance.

Degrees of Freedom — The number of motions associated with a dynamic system.

Shock Spectrum — The quantitative description of a shock motion, measured in terms of the response of a one-degree-of-freedom system to the shock motion.

Equivalent Static Acceleration — The maximum dynamic response of a dynamic system.

Excitation — The external force or motion applied to a system that causes the system to respond dynamically.

Sinusoidal Vibration — A vibratory motion, the amplitude of which is described by a sinusoidal function of time.

Mode Shape — A characteristic pattern assumed by the system in which every discrete point is vibrating with a sinusoidal motion at the same natural frequency.

Resonant Frequency — A frequency at which any sinusoidal excitation will produce responses significantly greater than the inputs. Also a frequency at which a system will vibrate after an excitation is applied and then removed.

Stiffness — The ratio of change of force (or torque) to the corresponding change in translational (or rotational) deflection of an elastic element of the system.

Shock — A transient, non-periodic, excitation.

Fundamental Frequency — The lowest resonant frequency of a system

Natural Frequency — A frequency of free vibration of a system.

Transmissibility Curve — A curve that relates the system response to the system exitation as a function of the exitation frequency.

Random Vibration — A vibratory motion, the instantaneous amplitude of which can be specified only on a probability basis.

Gaussian Distribution — The probability distribution which has been found to describe suitably the statistical distribution of the instantaneous magnitude of many structural random vibrations. (Bell-shaped curve)

Residual Response — Vibratory response of the system subsequent to termination of a shock pulse.

Yield Stress — The stress at which the stress-strain graph in a tension test deviates significantly from a straight line (usually at a strain of 0.002).

Modulus of Elasticity — The rate of change of uniaxial stress to strain, within the elastic range, in a tension test.

GLOSSARY (Continued)

Rupture Stress — The nominal stress developed in a material at rupture; not necessarily equal to the ultimate stress.

Buckling Stress — The stress (compressive) at which a structure collapses due to instability.

Stress Reversal — A cyclic loading condition in which tensile and compressive stresses alternate every half-cycle.

Cumulative Damage — The concept of fatigue failure where each cycle of stress imposes additional damage until total accumulated damage reaches 100%.

Endurance Limit — The maximum fluctuating stress a material can endure for an infinite number of cycles.

Allowable Stress — The maximum stress which may be applied to a structural element. (May contain a safety factor, at the option of whomever is responsible for establishing design criteria.)

Actual Stress — The stress resulting from the actual applied loads (perhaps modified by a safety factor).

Resonance Survey — A vibration test in which the frequency is slowly changed to determine resonances and where the excitation is low enough to prevent damage to the system.

Amplification Factor — A measure of the maximum transmissibility at resonance of a vibratory system, equal to one-half the reciprocal of the damping ratio for lightly damped systems. $(Q = 1/(2)\zeta)$

SYMBOLOGY

The following symbols will be used as standard throughout this volume:

Q	Amplification factor
ζ	Damping factor
TR	Transmissibility Ratio
f	Frequency, Hz
f_n	Natural frequency, Hz
PSD	Power spectral density, g^2/Hz
t .	Time, sec.; thickness, in.
rms	Root mean square
k	Stiffness, lbs/in
m	Mass, lbs-sec.^{2}/in
δ, Δ	Deflection, in
W	Weight, lbs.
ρ, r	Radius of gyration, in.
E	Modulus of elasticity, lbs/in^2
S	Stress, lbs/in^2
ϵ	Strain
G_{se}	Static equivalent acceleration
L	Length, in.
R	Radius, in.
S_e	Endurance limit, lbs/in^2
MS	Margin of safety
τ	Shear stress, lbs/in^2
γ	Shear strain
G	Shearing modulus of elasticity, lbs/in^2
P	Load, lbs
g	Acceleration due to gravity, in/sec^2
b,h	Length of plates sides, in.
A	Area, in.^{2}
p	Pressure, lbs/in^2
D	Plate flexural stiffness, lb.-in.
ω	Circular frequency, rad/sec.
μ	Poisson's Ratio

DROP TEST SHOCK RESPONSE

The response of electronic equipment to the prescribed drop testing is computed on the assumption that the equipment can be idealized as a one-degree-of-freedom dynamic system. The physical system and its analytical model are illustrated in the accompanying diagram. The response of the mass to the impact shock is computed as follows. The potential energy of the mass at the instant of release is mgh. After release, this is converted to kinetic energy which is maximum at the instant of impact. This kinetic energy is then converted to strain energy as the spring deforms. In terms of maximum spring deformation X, the strain energy is $1/2 \, kX^2$. Equating the maximum strain energy to the maximum potential energy yields:

$$1/2 \, kX^2 = mgh$$

Thus the maximum spring deflection is:

$$X = \sqrt{2 \frac{mgh}{k}}$$

The corresponding acceleration is given by the force (kX) divided by the mass (m):

$$\ddot{X} = \frac{kX}{m} = \frac{k}{m} \sqrt{2\left(\frac{m}{k}\right)gh} = \sqrt{2\left(\frac{k}{m}\right)gh}$$

In terms of "g's":

$$G_{se} = \frac{\ddot{X}}{g} = \frac{1}{g} \sqrt{2\left(\frac{k}{m}\right)gh} = \sqrt{\frac{k}{m}} \sqrt{\frac{2h}{g}}$$

$$G_{se} = 2\pi f_n \sqrt{\frac{2h}{g}} = 0.452 \, f_n \sqrt{h}$$

Drop heights (h) of 12, 24, 30, 36, and 48 inches are specified for various equipments, as detailed in the accompanying table. Thus, a separate shock spectrum curve can be drawn for each value of h. These are shown in the accompanying graph.

PHYSICAL SYSTEM

Carrying Case

Equipment

Isolators (optional)

Drop Height (h)

2" Fir

Concrete or Steel

TEST PARAMETERS

MODEL

Weight	Size	Drop Height, (h) (in.)
Under 100	Under 36	48
	Over 36	30
100-200	Under 36	30
	Over 36	24
	Under 36	24
Over 200	36-60	36
	Over 60	24
Over 1000	Over 60	12

SHOCK SPECTRUM
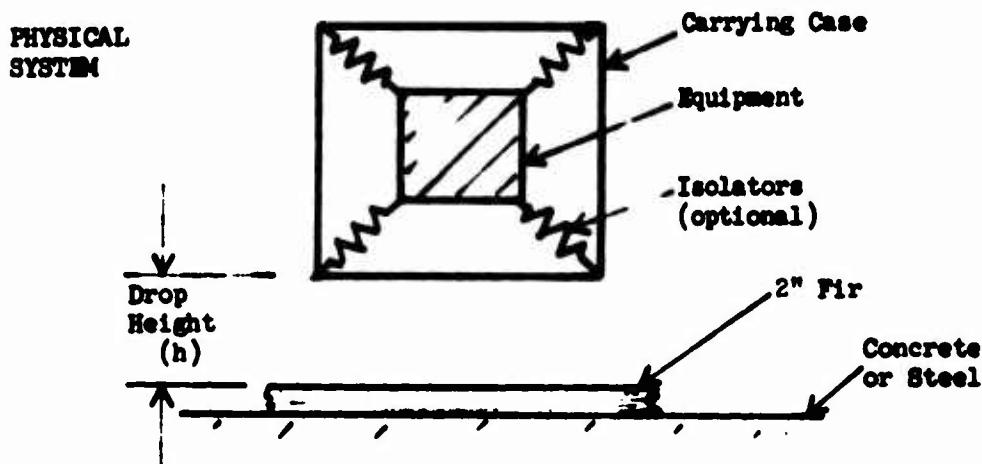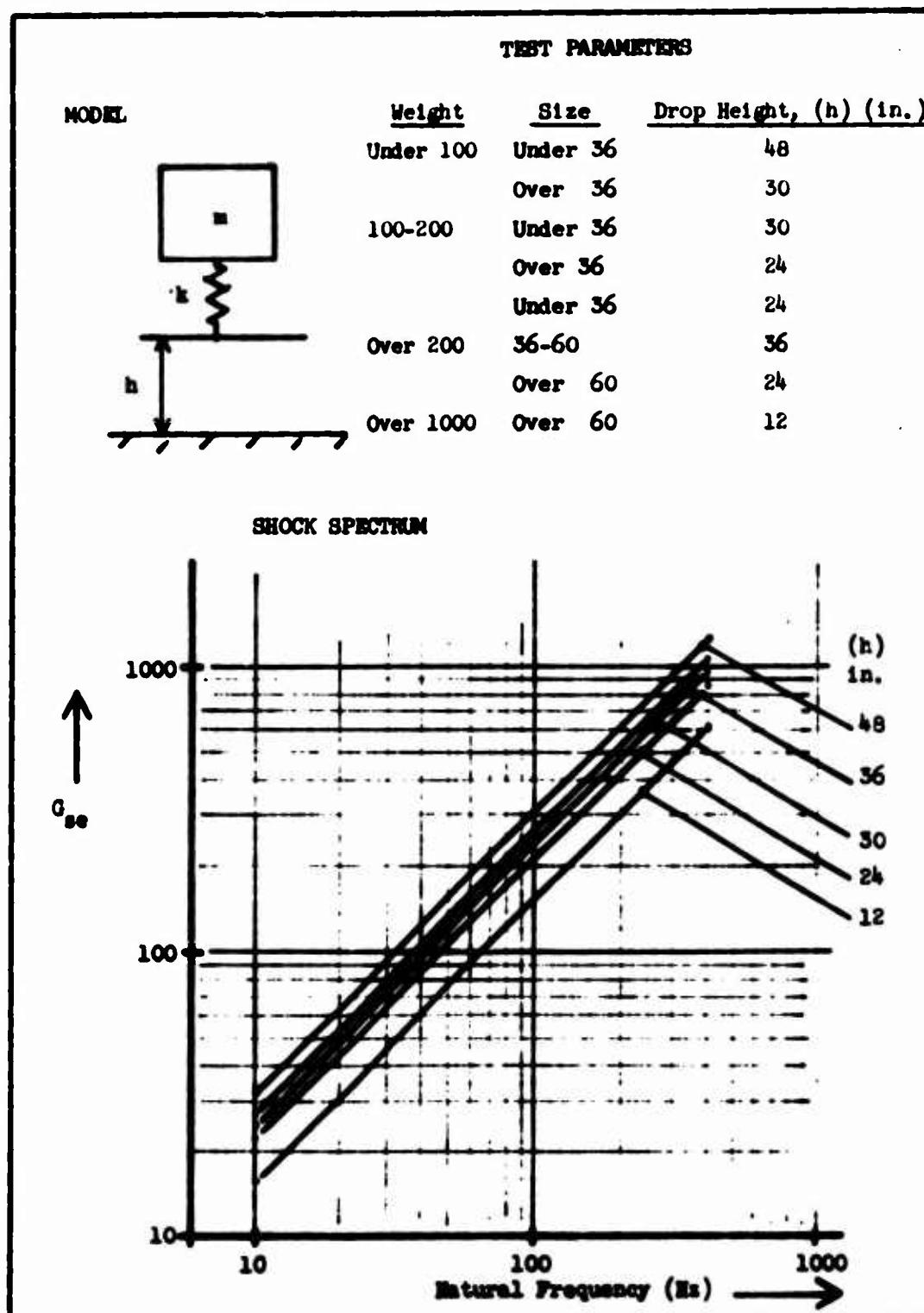
DROP TEST: Responses depend upon drop heights and natural frequencies.

BENCH HANDLING SHOCK RESPONSE

The bench handling tests consist of raising the equipment above a test
bench, while one edge is kept in contact with the bench, and then releasing
the equipment and allowing it to fall to the bench. The configuration is
shown in the accompanying diagram. The maximum height required is four
inches, so that this test is much less severe than the drop test. There-
fore, no attempt is made to generate shock spectra corresponding to all
angles and values of h. Rather, only the maximum value of four inches
'll be considered.

Th. analytical model for this case consists of a one-degree-of-freedom
sys which is dropped from a height of 1/2 h. This model neglects the
rotat al inertia of the equipment. However, this leads to a conserva-
tive r 't because the rotational inertia would decrease the velocity at
impact, us calling for a smaller effective drop height.

The dyna c behavior of this system was derived in the preceding topic;
where it shown that the response acceleration is:

$$G_{se} = \cdot \quad f_n \times \sqrt{\text{drop height}}$$

In this case the drop height is 1/2 h, so that:

$$G_{se} = 0.452 f_n \sqrt{\tfrac{1}{2} h} = 0.32 f_n \sqrt{h}$$

for h = 4 inches, the response is:

$$G_{se} = 0.32 f_n \sqrt{4} = 0.64 f_n$$

The corresponding shock spectrum is shown in the accompanying diagram.

PHYSICAL SYSTEM

Equipment

Table

θ

h/2

h

MODEL

1/2 h

SHOCK SPECTRUM

a_{se}

Natural Frequency (Hz)
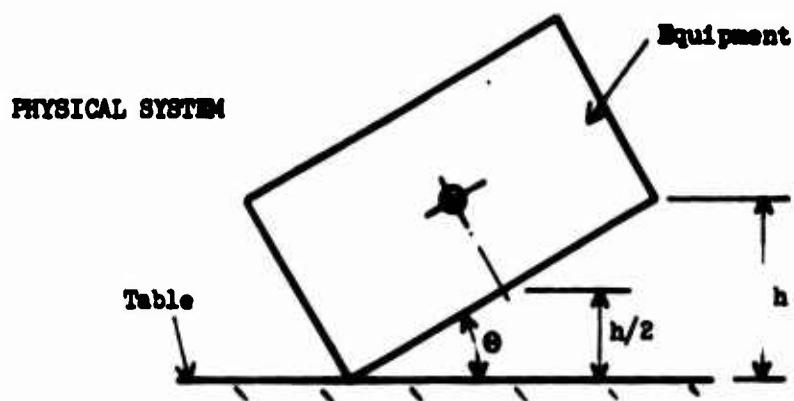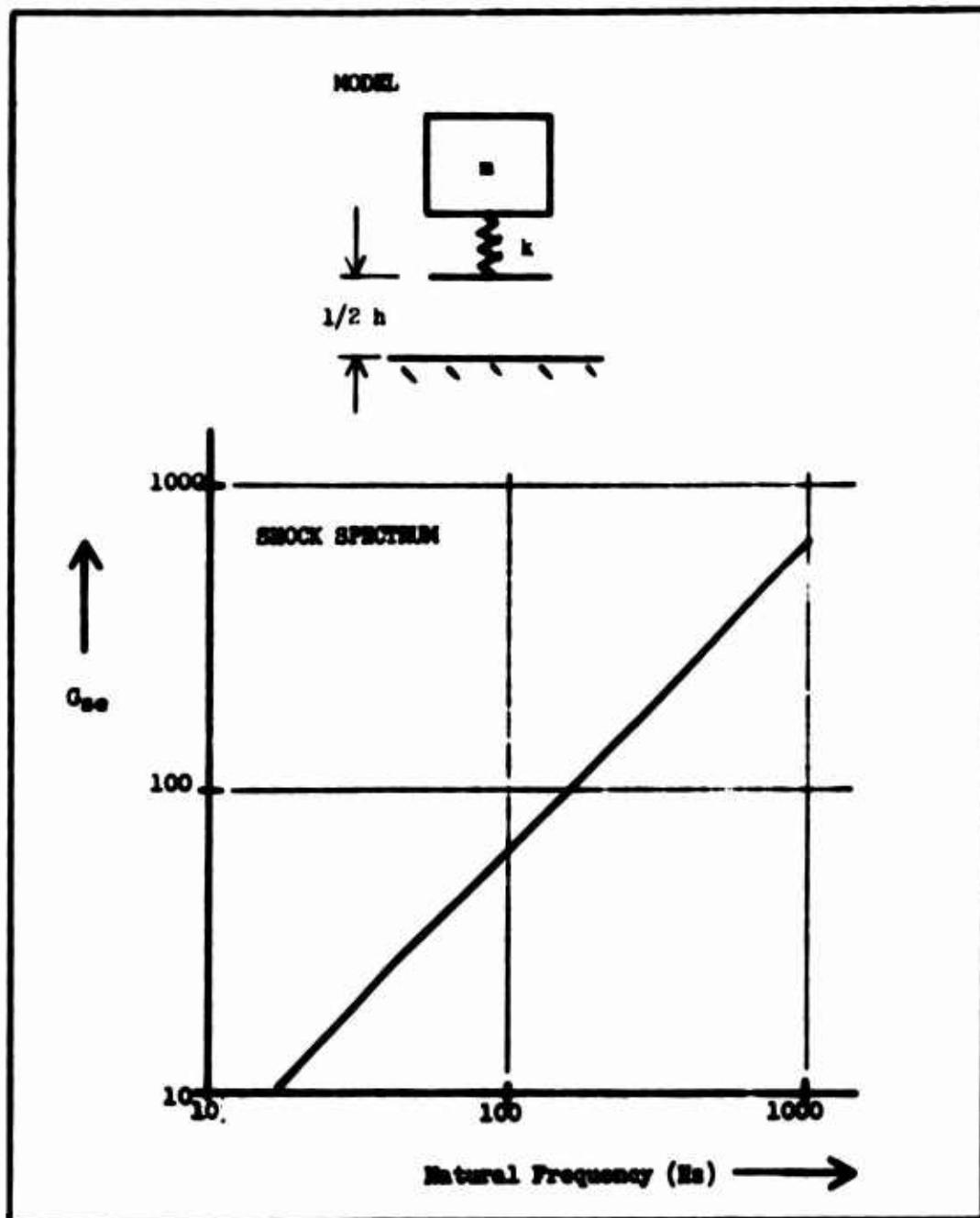
BENCH HANDLING: Responses are computed on the basis of an assumed drop height of 4 inches.

END
11 - 70